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LXIV. *The Constitution of the Upper Strata of the Atmosphere.* *By* L. VEGARD, *D.Sc., Professor of Physics at the University of Christiania**.

[Plate X.]

Auroral Spectra at different Altitudes.

§ 1. IN a communication recently published † I gave the results of investigations on the auroral spectrum carried out during last winter. Since I wrote this paper I have, during a stay at Tromsø from the end of February to the end of March, been able to carry out further observations on the auroral spectrum, and above all I have succeeded in taking spectrograms corresponding to various altitudes. At the same time I have continued investigations on the distribution of gas density in an electrically-charged atmospheric layer.

These investigations have confirmed the view already announced in my first paper with regard to the substances present in the higher strata of the atmosphere and the origin of the green auroral line, but with regard to the physical state of these strata they have led to a modification of the view announced in my first paper.

In this supplementary communication I am going to give a short account of the results of these investigations.

In the first paper we saw that most of the lines appearing in the auroral spectrum are due to nitrogen, but the strongest

* Communicated by the Author.

† Phil. Mag. July 1923, p. 193.

of all lines (the green line) and three faint lines in the blue and the ultra-violet could not be found among the lines of nitrogen observed in laboratory experiments and given in the tables of the spectral lines.

Nor can they be found among the known lines of hydrogen and helium. Already this result suggests that also the green line and the other three not identified lines are due to nitrogen.

The spectral lines given in my first paper were photographed by directing the instrument towards the strongest parts of the auroræ, and the spectra thus obtained will be mainly determined by the light emitted in a height interval near the bottom edge between 100 and 120 km. say. In this interval nitrogen is found to be a predominant component, and in this interval also the green line appears with a very great intensity.

If the green line is emitted from some gas which is different from nitrogen and which has not entered into any kind of combination with this element we should expect that the intensity of the green line relative to that of the nitrogen bands would change rapidly as we pass upwards from the bottom edge of an auroral streamer.

By means of a simple spectroscope I have examined the light emitted from various parts on very long auroral rays, and it appears that the green line is emitted to the very extreme upper limit of rays reaching altitudes of several hundred kilometres. The nitrogen bands have a very small effect on the eye and a very strong aurora is required to make them visible in a spectroscope, and in the case of long ray streamers the N-bands are usually not noticeable. We are thus under the necessity of using a photographic method for the solution of our problem.

For this purpose we want a spectrograph of the very greatest light power, but as we are only studying intensity variations we need no great dispersion. Already in the first paper I have mentioned that a small spectrograph of great light power was constructed for studying the possible variations of the auroral spectrum (spectrograph nr. III.).

Before I went to Tromsø this winter I had a second spectrograph of the same type made, and in this way I was able to work with two spectrographs simultaneously. It will be of interest in this connexion to give a short description of these instruments.

In the construction of the instrument I made use of the fact that the light power of a spectrograph does not depend on the light power of the collimator lens, provided that the

camera lens has a great light power. The smaller light power of the collimator lens is compensated by using a wide slit.

The construction of spectrographs on this principle has the following advantages:—

(1) The effective cone of the instrument can be filled with light from a source of light which has fairly small angular dimensions seen from the slit.
(2) A lens of small light power is cheaper and easier to make.
(3) As the picture of the slit is much reduced the slit need not be so exactly made to give equally good lines on the picture.

The collimator lens had a diameter of 3 cm. and a focal distance of 18 cm. The camera lens had an effective opening of 2·5 cm. and a 5 cm. focal distance, and thus the light power was 1 : 2. Rutherford prisms were used in both spectrographs.

On the collimator could be fixed a cylindrical tube carrying a lens with about the same aperture and focal distance as the collimator lens. When the instrument was directed towards an aurora, this lens formed a picture which fell on the slit. In this way we ensure that when the optical axis of the collimator is directed on a certain point of an aurora, only light from the nearest surroundings of this direction can enter the instrument. In other words, we are able to analyse separately different parts of an auroral display.

During the evenings and nights of March 11th and 12th I succeeded in taking two pairs of such photograms, one pair each day. The spectrogram corresponding to the bottom edge was obtained with about half an hour's exposure. Near the upper limit the light is fainter and the spectrum was here exposed for about four hours. Still the intensities on the plate of the two spectra were nearly the same. Reproductions of the two pairs of spectrograms are given on Plate X. figs. 1 and 2 for March 11th and 12th respectively:

a is the spectrum corresponding to the bottom edge.
b „ „ „ upper limit.

The auroræ used for these exposures were mostly drapery-shaped arcs and some draperies and isolated rays. The bottom edge corresponds to a height of about 100 km. and the two spectra to be compared correspond to a difference of height of about 50–80 km.

The intensities of the spectral lines will be accurately

measured later on by means of a microphotometer, but some results can be directly seen on the plates.

Thus the lines belonging to the line spectrum of nitrogen compared with those of the negative bands seem to be somewhat stronger near the upper limit than near the bottom edge, but the most remarkable result is, that *the intensity of the green line as compared with that of the strong negative N-bands is quite as great near the bottom edge as it is* 50–80 *km. higher up in the atmosphere.*

If there is a change of relative intensity it is very small indeed, and goes in such a direction as to make the negative N-bands relatively stronger as we pass upwards.

Discussion and Results.

§ 2. In order to make clear the significance of this result we assume the upper strata of the atmosphere to be composed of Nitrogen, Oxygen, Hydrogen, Helium, and Geocoronium in the way given by Wegener, and we shall calculate the ratio of the pressures of H, He, and O to that of nitrogen for various heights. The results are given in Table I. :—

TABLE I.

Height. km.	H/N.	He/N.	O/N.
80	2·7	0·18	0·049
100	45	2·4	0·032
120	760	33·0	0·025

In the case of "Geocoronium," which is supposed to be a gas lighter than hydrogen, the ratio Geoc./N should vary even more than with hydrogen.

It appears from the table that only a comparatively small difference of molecular weight such as that of O and N would produce a considerable change with altitude of the ratio between the pressures of the two gases.

Comparing the numbers of Table I. with the results of our spectral analysis it follows that the green line cannot *be due to any elementary gas different from nitrogen*, provided that the matter in the auroral region exists in a gaseous state. The green line must either be emitted from nitrogen itself or from some substance which in these regions has entered into combination with nitrogen. Here merely compounds with elements lighter than nitrogen can come into consideration, because a compound formed with some heavier element could not keep its place on the top of the ordinary nitrogen atmosphere.

Briefly speaking, of known compounds only NH_3 could come into consideration. But if the upper strata were composed of NH_3 we should expect that the auroral spectrum should show the ordinary hydrogen lines with considerable intensity. But as we saw in the first paper this is not the case. The green line cannot be due to a hypothetical gas "Geocoronium" if this gas exists in the elementary form, but we should have to suppose that this gas had a great chemical affinity to nitrogen and thus formed a compound with this gas. An assumption of this kind would quite formally agree with the results of our analysis of the auroral spectrum. It is against our present knowledge with regard to the elements constituting matter, and I think that the "Geocoronium" hypothesis ought to disappear from the scientific literature, and if so the only possible interpretation of our auroral spectral analysis is to assume that also the green line is a nitrogen line emitted under those peculiar physical conditions under which the light is produced in the upper strata of the atmosphere. Later on we shall see what these conditions may possibly be.

As already mentioned the green line is observed to the upper limit of very high rays reaching altitudes of 300–400 km. or more; and as it appears that the ratio between the intensity of the green line and that of the nitrogen lines does not increase as we pass upwards, *it follows, independent of the view we adopt with regard to the origin of the green line, that nitrogen must be a predominant component of the atmosphere to its very extreme upper limit.*

Already in my first communication I was led to the same view, and I gave several arguments in favour of it, but at the same time I mentioned that the correctness of it might be tested by comparing auroral spectra corresponding to different altitudes, and as we see this comparison has shown that the view adopted was the right one.

In the first paper I discussed the consequences of the view, which now is to be regarded as an established fact, that nitrogen is a predominant component of the atmosphere as far up as noticeable auroral light can be observed.

To explain that nitrogen at a height of 500–700 km. could have a sufficient density to give noticeable light emission, we might either assume that above a certain height the temperature increased rapidly upwards or that the nitrogen was electrically charged by photo-electric effect and driven upwards through the effect of the electric field produced by the charged layer.

The assumption of a very high temperature of several

hundred degrees centigrade might quite formally explain the relatively great density of N in the auroral region, but is very unlikely from a physical point of view; and above all such a state of the atmosphere cannot explain the peculiar type of nitrogen spectrum which is emitted from these regions during auroral displays.

An increase of temperature would involve that the amount of matter to be traversed by the cosmic electric rays in order to reach a distance of 100 km. from the ground would be very much greater than the amount of nitrogen which according to Wegener's assumptions should exist above this level.

Referring to earlier investigations with regard to the properties of the rays which may produce the auroræ, we can draw the conclusion that only electron rays could produce the auroræ—provided that we assume a high temperature to exist in the auroral region.

In this case the auroral spectrum, the green line included, should be produced when nitrogen gas of a temperature of some hundred degrees centigrade is bombarded by cathode rays.

In order to test this point I have photographed the spectrum emitted when nitrogen is bombarded by cathode rays at room temperature and at the temperature of liquid air. The result will be seen from Pl. X. fig. 3, where 3*a* corresponds to room temperature and 3*b* to the temperature of liquid air.

In fig. 4 of the same plate is given a reproduction of the auroral spectrum obtained by the big glass spectrograph (4*b*) and a reproduction of a spectrum from the quartz spectrograph (4*a*).

Apart from the green line, the appearance of which will be discussed later on, the auroral nitrogen spectrum is distinguished by a relatively small number of lines—and although most lines form part of positive or negative bands the greater part of the lines appears fairly sharp and the shades towards shorter wave-lengths are very much suppressed.

Comparing the two N-spectra at different temperatures we see that an increase of temperature is accompanied by an increase of the number of lines and the shades of the bands (number of components in the band series) are also increased with temperature.

While there is a striking similarity between the auroral spectrum and that corresponding to the temperature of liquid air, this similarity is by no means so marked when we compare the auroral spectrum with that corresponding to

room temperature, and when we assume a high temperature in the auroral region there seems to be no possible way open for explaining the appearance of the green line.

On the Distribution of Matter in an Electrified Atmosphere.

§ 3. *If now we have to give up the assumption of a high temperature I can see no escape from the assumption that nitrogen in some way or other must be driven upwards through the effect of electric forces.*

This hypothesis introduces the problem of finding how the density of an electrified atmosphere will vary with the height as we pass upwards from the neutral part of the atmosphere.

From the phenomenon of electric wind we are familiar with the idea that an electrically charged gas in an electric field is exposed to the action of a mechanical force, and already in the first paper I assumed that the variation of pressure was given by the equation

$$dp = -(\rho g - \sigma \mathrm{F})dr, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where p is the pressure, ρ the density, σ the electric charge per unit volume of the gas, F the electric force, and r the distance from the centre of the earth to the point considered.

From this equation we see that the electric charge will have the effect of diminishing the rate of decrease of the density as we pass upwards. If we try to calculate the distribution of matter more accurately we shall soon find that the problem is a somewhat complicated one.

It is my intention to show that under certain assumptions we can find a differential equation for the variation of density with the height, and that we are able to draw some consequences from our hypothesis of an electrically charged upper layer.

Let us suppose the earth to be a perfect sphere and that ρ and σ are functions of r only. We apply the theorem of Gauss to a spherical surface through the point P at a distance r from the centre of the earth. Then

$$4\pi r^2 \mathrm{F} = 4\pi \mathrm{E}_r,$$

$$\mathrm{F} = \frac{\mathrm{E}_r}{r^2}.$$

The total electric charge E_r inside the sphere is given by

the equation :

$$E_r = E_0 + 4\pi \int_{r_0}^{r} \sigma r^2 dr,$$

where E_0 is the total charge inside some arbitrary sphere with radius r_0. Then

$$F = \frac{E_0}{r^2} + \frac{4\pi}{r^2} \int_{r_0}^{r} \sigma r^2 dr. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

We suppose that the usual relation between pressure and density of a gas also holds in an electric atmosphere, or that

$$p = \frac{RT}{M} \rho, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where M is the molecular weight and R the gas constant.

Combining equations (1), (2), and (3) we get :

$$\frac{RT}{M} d\rho = -\left[\rho g - \frac{\sigma E_0}{r^2} - \frac{4\pi\sigma}{r^2} \int_{r_0}^{r} \sigma r^2 dr\right] dr.$$

Putting $\frac{RT}{M} = a$,

$$\frac{ar^2}{\sigma} \frac{d\rho}{dr} + \frac{\rho g r^2}{\sigma} - E_0 = 4\pi \int_{r_0}^{r} \sigma r^2 dr. \quad . \quad . \quad . \quad (4)$$

Differentiating with respect to r and remembering that ρ and σ are functions of r, the equation takes the form :

$$\frac{1}{\sigma} \frac{d^2\rho}{dr^2} - \frac{1}{\sigma^2} \frac{d\sigma}{dr} \frac{d\rho}{dr} + \left(\frac{2}{r} + \frac{g}{a}\right) \frac{1}{\sigma} \frac{d\rho}{dr} - \frac{g\rho}{a\sigma^2} \frac{d\sigma}{dr} + \frac{2g\rho}{a\sigma r} - \frac{4\pi\sigma}{a} = 0.$$

$$. \quad . \quad . \quad (5a)$$

The equation contains two unknown quantities ρ and σ, and to solve the problem one equation more is wanted.

This second relation depends on the way in which the atmosphere is brought into its charged state.

In the previous papers it was pointed out that the atmosphere might be positively charged through the photo-electric effect (secondary cathode rays) produced by X- or γ-rays from the sun, but even if this is the only cause of the charged state of the atmosphere we are not in the possession of sufficient data to find the charge given to the atmosphere in this way. Whatever may be the cause of the charged state, we must assume the charge attached to the gas. Thus σ may

be regarded as a function of ρ and the equation (5) takes the form:

$$\frac{1}{\sigma}\frac{d^2\rho}{dr^2} - \frac{d\sigma}{d\rho}\left(\frac{1}{\sigma}\frac{d\rho}{dr}\right)^2 + \left(\frac{2}{r} + \frac{g}{a} - \frac{g\rho}{a\sigma}\frac{d\sigma}{d\rho}\right)\frac{1}{\sigma}\frac{d\rho}{dr} + \frac{2g\rho}{ar\sigma} - \frac{4\pi\sigma}{a} = 0. \quad . \quad . \quad . \quad (5b)$$

Now we may always put

$$\sigma = \alpha\rho,$$

where α is a function of ρ. And we know that in a great many cases the auroræ have the same colour from the upper limit to the bottom edge. If the auroral region is electrically charged this would indicate that in this region α should be nearly a constant quantity. We are thus led to consider the case that α is constant down to a certain distance from the centre r_0.

Putting this distance equivalent to r_0 in equation (2) it follows that above this distance the density should be determined by the differential equation:

$$\frac{d^2\rho}{dr^2} - \frac{1}{\rho}\left(\frac{d\rho}{dr}\right)^2 + \frac{2}{r}\frac{d\rho}{dr} + \frac{2g\rho}{ar} - \frac{4\pi\alpha^2\rho^2}{a} = 0. \quad . \quad . \quad (6)$$

Even under these simplified conditions the differential equation is a rather complicated one, and up to the present I have not been able to find the integral function. But at any rate, we may find the variation of ρ by means of numerical integration.

On the Possibility of a highly electrified Gas Layer.

§ 4. Without entering further into the problem of solving the differential equation, we can make some estimates as to the degree of ionization and the amount of electricity which may be present in the higher strata of the atmosphere, and draw some consequences of importance with regard to their state.

We know that the earth—in some way or other—maintains a negative charge, and this charge will be compensated by positive electricity in the atmosphere. This charging process must be continually going on, and we might perhaps suppose that the positive electricity is formed by photo-electric action in the upper layer of the atmosphere. At a certain height the positive electric mass just equals the negative, and on a sphere at this distance the electric force will be zero. Under this niveau the electric force is directed downwards, and

only above it will there be an upwardly directed electric force.

It is only the positive electricity above this zero layer that will be responsible for the electric field of the earth outside the zero sphere.

If the sun emits X- or γ-rays, these rays, when they strike the upper strata of the atmosphere, will cause the gas to emit electrons with a velocity given by Einstein's equation :

$$h\nu = \tfrac{1}{2}mv^2 = e\mathrm{V}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Some of these electrons leave the earth, and this process will continue until the potential of the earth V is equal to the potential which is determined from the frequency.

If the rays had a wave-length of 1 Å. the potential should be 41 electrostatic units.

At the very limit of the atmosphere, *e. g.*, 600 km. above the ground, there should be an electric force :

$$\mathrm{F} = \frac{\mathrm{V}}{\mathrm{R}} = 6 \,.\, 10^{-8} \text{ el.st. units.} \quad . \quad . \quad . \quad (8)$$

Probably there may be a radiation of much shorter wave-length and the electric force would be correspondingly greater.

Now, if we have a nitrogen ion with a positive charge $+e$, at the same point, it will be driven upwards with a force Fe and downwards with the force mg. If the ion does not leave the earth we must have

$$mg > \mathrm{F} \,.\, e.$$

Putting

$$m = \mathrm{M}m_{\mathrm{H}},$$

where m_{H} is the mass of a hydrogen atom

$$\mathrm{F} < \frac{g\mathrm{M}}{3 \,.\, 10^{14}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If the ion is a charged molecule M = 28, and

$$\mathrm{F} < 10^{-10} \text{ el.st. units.}$$

We see that only an extremely small electric force would be sufficient to counterbalance gravity, and the electric force which might be produced through the electron emission due to photo-electric effect is several hundred times larger than that which is necessary to drive a charged molecule away from the earth.

It seems as if the emission of X- and γ-rays from the sun would gradually tap the atmosphere of its nitrogen. This

tapping must be counterbalanced in some way, and the idea naturally suggests itself that the ions are not charged molecules, but at the very low temperature they form clusters consisting of a large number of molecules.

We can also arrive at the same conclusion in another way, by calculating the mass per unit area above the zero sphere. We suppose a cylinder of unit cross-section placed radially and with one end at the surface of the zero sphere and the other reaching outside the atmosphere. Let the mass contained in this cylinder be μ and the number of molecules n. Then

$$\mu = nm.$$

Let us suppose that a fraction β of these molecules carry an elementary charge e. The total charge ϵ in the cylinder would then be

$$\epsilon = \beta ne.$$

Hence

$$\epsilon = \beta \frac{e}{m} \mu. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

As the length of the cylinder is small as compared with the radius of the earth the total electric charge above the zero sphere should be

$$\mathrm{E} = 4\pi r^2 \epsilon,$$

and just at the limit of the atmosphere the electric force should be

$$\mathrm{F} = \frac{\mathrm{E}}{r^2}, \quad \text{or approximately}$$

$$\mathrm{F} = 4\pi\epsilon = 4\pi\beta \frac{e}{m} \mu. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

If at the outer limit of the atmosphere the action of the gravitation on unit volume is greater than the mechanical force due to the electric charge we must have:

$$\rho g > \sigma \, . \, \mathrm{F}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Let the number of molecules in unit volume be n', then

$$\rho = mn',$$

$$\sigma = \beta' n' e, \quad \text{and}$$

$$\frac{\sigma}{\rho} = \beta' \frac{e}{m}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

β' is the ratio of the number of charged molecules to the

total number in unit volume. β' may vary with r, and thus be different from β, which is an average for the region above a certain level. To get an estimate we will put $\beta' = \beta$.

Inserting the expressions for $\frac{\sigma}{\rho}$ and F, the condition (12) takes the form

$$g > 4\pi\mu\beta^2\left(\frac{e}{m}\right)^2.$$

Now,

$$\frac{e}{m} = 1{\cdot}07 \,.\, 10^{13}, \text{ and}$$

$$4\pi\beta^2\mu < 10^{-23} \text{ gram.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

If each molecule carried a charge e, $\beta = 1$ and $4\pi\mu = 10^{-23}$ gram. This would mean that practically no charged matter could exist above the zero sphere. Underneath the zero sphere, however, the electric force is directed downwards, and the electric charge could not account for the density of nitrogen diminishing as slowly as would be necessary to explain the very long auroral ray streamers.

If the upper strata of the atmosphere were in an electrically charged state we should have to assume that β is very small. But at the very low densities it is rather doubtful whether an individual ion of molecular dimensions will impart to the other molecules a mechanical force as if the electric charge had been equally divided on all molecules.

If we shall stick to the hypothesis that nitrogen maintains a noticeable density to heights of 500–600 km. through the effect of electric forces, we must assume that the charged particles form clusters of drops of a very small size. If the temperature is extremely low it seems possible that nitrogen might condense round the ions. We know that the freezing-point of nitrogen is at about 60° Abs., and if the temperature were still lower nitrogen might be present in the form of minute crystals, or fine solid dust.

If the diameters of the drops or dust-particles were of the same order of magnitude as the wave-length of light there might be about 10^8 molecules in each particle. In equation (9) we should then have to put $M = 28 \,.\, 10^8$, and in order to prevent the dust-particle from leaving the earth it is only required that:

$$F < 10^{-2} \text{ electrostatic unit,}$$

and this force is much greater than that which would result from photo-electric action from the sun provided the maximum frequency of the rays was 1 Å.

On the Amount of Matter in the Dust Layer.

§ 5. If we assume this physical state of the upper strata, we are left rather in uncertainty with regard to the amount of matter which may be present in the auroral region. We can no longer apply the gas laws as expressed in the equations (1) and (3), and the variation of average density of the dust layer is not determined by the differential equation (5).

The dust-particles will be of unequal size and charge, and according as $\frac{e}{m}F$ is greater or smaller than g, the particle will move upwards or downwards. A particle that is moving upwards may pick up an electron and thus get its positive charge reduced so that it begins to sink. We have also to reckon with the possibility that some particles that are moving upwards do not pick up electrons and thus leave the earth, and the question arises: is this possibility in some way excluded or is nitrogen in minute quantities continually leaving the earth, and if so, is the nitrogen which is lost replaced by new supplies coming in from space—*e. g.*, from the sun? At present I merely put these questions, which can give room for many speculations and suggestive ideas.

The sinking particles may, some of them, lose electrons and can be caused to move upwards, others may continue to sink till they get vaporized. To re-establish equilibrium equal quantities of gas will move into the dust atmosphere and condense into small drops or crystals.

The density of matter in the gas atmosphere may be approximately estimated. How much matter can be kept up by electric forces, however, is not easily found. It will depend on the intensity and wave-length of those sun-rays which are mainly responsible for the photo-electric action and the ionization; further, the amount of matter will depend on the height of the transition layer between gas and dust atmosphere. From auroral observations we can conclude that this height is less than 100 km.

Through our investigations we are led to a conception with regard to the constitution of the higher strata of the atmosphere fundamentally different from the view earlier adopted among scientists. It has hitherto always been supposed that there was an upper layer of light gases He and H. The composition of the atmosphere according to the

older view is illustrated in fig. 1. The curves are drawn in accordance with Wegener's assumptions. Various authorities differ with regard to temperature and amount of hydrogen and helium, but in any case we should get a distribution of the type given in fig. 1.

Fig. 1.

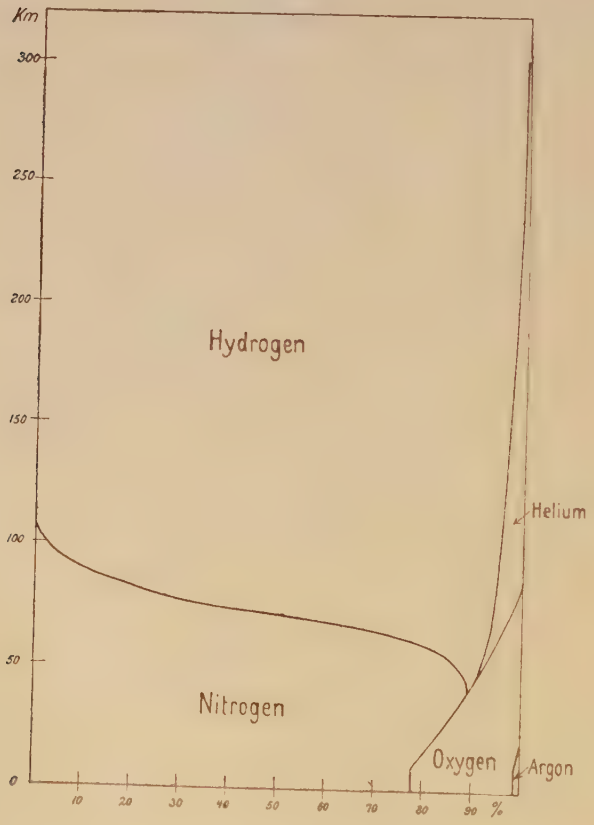
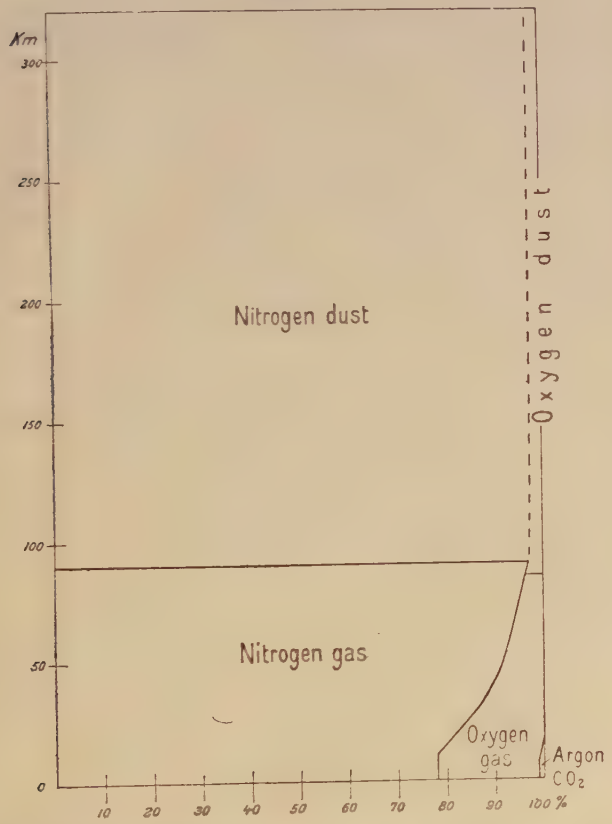


Fig. 2 gives a similar representation of the composition of the atmosphere here found from our investigations on the auroral spectrum. The transition between gas- and dust-atmosphere is indicated at a height of 90 km.

Our view with regard to the physical state of the upper strata of the atmosphere makes it at once intelligible that there can be no hydrogen and helium layer on the top of our atmosphere, for on account of the much lower freezing-point

of the gases they may be unable to form clusters or dust-particles, and as soon as a hydrogen molecule or a helium atom reaches above the sphere of zero electric force it will sooner or later get ionized and fly out of the atmosphere as a positive ray. The distribution of light intensity of the

Fig. 2.



various auroral forms seems to indicate that the dust atmosphere has a comparatively great density in the height interval 100–120 km., but above this interval the average density seems to be very small, but the existence of long auroral rays keepinga nearly constant intensity for a distance of several hundred kilometres shows that the average density in the dust layer above 120 km. only decreases very slowly as we pass upwards.

The Nature of the Cosmic Electric Rays.

§ 6. As we are left very much in uncertainty with regard to the amount of matter present in the auroral region we are also left in doubt as to the nature of the rays that produce the auroræ. From the study of the variation of light intensity along the auroral streamers I was able to show * that the electric rays do not move in nearly straight line tracks but that they must move in spirals round the magnetic lines of force.

From measurements of the thickness of the auroral streamers we then found that the cosmic electric rays had to fulfil the conditions

$$\frac{mv}{e} \leqq 10^4. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Further, we found the range x in nitrogen at atmospheric pressure to be approximately given by the equation :

$$x = 10^{-27} \frac{M}{n^2} v^3 \text{ cm.} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The ray carrier is supposed to have a mass M/N and a charge ne, where N is Avogadro's number, and e the elementary charge.

If now the rays fulfil the equation (15) we get

$$x \leqq 10^{-3} \frac{n}{M^2} \text{ cm.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Taking into account our knowledge with regard to the constitution of matter $\frac{n}{M^2}$ will decrease with increasing mass of the carrier. The amount of nitrogen which, according to the assumptions of Wegener, had to be traversed by a ray reaching a height of 100 km. is equivalent to a layer of 0·2 cm. reduced to atmospheric pressure. In that case only rays of negative electrons could, according to equation (17), give a sufficient penetrating power. For hydrogen nuclei $n = 1$ and $M = 1$, and equation (17) gives $x \leqq 10^{-3}$ cm., or a hydrogen ray having the sufficient magnetic deflectibility could not reach a height of 100 km., provided that the amount of nitrogen above 100 km. is that assumed by Wegener †.

It will now be understood that since our results with

* See L. Vegard, *Forhandlinger ved Skandinaviske Geophysikermøtet i Göteborg*, 28–31 Aug., 1918, p. 12 ; Vegard and Krogness, Geophys. Publ. vol. i. no. 1, p. 159 ; Vegard, Phil. Mag. vol. xlii. p. 59 & p. 79 (1921).

† See L. Vegard, Phil. Mag. xlii. Table X. (1921).

regard to the constitution of the upper strata of the atmosphere leave us in doubt with regard to the amount of nitrogen present in the auroral region we cannot conclude that the auroræ are necessarily produced by electron rays.

In any further treatment of the problem we should have to take into account that the absorption does not any longer take place in a gas or in a system which may be regarded as homogeneous. We must, in other words, remember that even if an electric ray is absorbed by a single dust-particle it may have a fairly good chance of getting far down into the dust-layer when the average distance between the particles is fairly large.

Thus we see that the question with regard to the nature of the cosmic rays has come into a new position through the results of our analysis of the auroral spectrum.

The Auroral Spectrum.

§ 7. According to our view with regard to the physical state of the atmosphere, the auroral spectrum should be formed when electric rays penetrate into an atmosphere mainly consisting of small nitrogen crystals which are more or less electrically charged. As each particle consists of a great many molecules only carrying a few elementary charges the spectrum emitted cannot depend essentially on the electric charge. But if these are the conditions under which the auroral spectrum is formed we can understand why a nitrogen spectrum like that given out by the auroræ has never been reproduced in laboratories, for we should then have to bombard solid nitrogen dust with electric rays.

It should be possible to reproduce, at any rate approximately, the conditions of the auroral light emission by cooling nitrogen by means of liquid hydrogen and bombarding the solid nitrogen by canal rays or cathode rays. At the present time I am making preparations for experiments of this kind.

When we consider the upper strata of the atmosphere to consist of dust particles, it might seem possible that also dust-particles of other kinds of matter might be mixed up with the nitrogen particles, and in that way there might be given a possibility of explaining the green auroral line. But such an explanation of the green line is not possible. For on account of the great intensity of the green line, the matter from which it is emitted must be present in relatively large quantities. As hydrogen and helium are excluded, these particles should be formed by some substance with an

atomic weight greater than that of nitrogen. Now some of these particles may lose their charge to such an extent that they sink below the sphere of zero electric force and then they will continue to sink.

Although minute quantities of cosmic dust may exist in the atmosphere we cannot assume them to be present at any height-interval of the auroral region in quantities almost exactly proportional to the number of nitrogen particles. Or the amount of cosmic dust per unit volume at a certain height in the auroral region should be

$$\rho_k(h) = \mathrm{K}\rho_N(h),$$

where $\rho_N(h)$ is the average density of nitrogen at the height h, and if the cosmic dust is responsible for the auroral line, K should be nearly independent of h and $\rho_k(h)$ and $\rho_N(h)$ should be of the same order of magnitude. But even if we assumed a constant "current" of cosmic dust falling towards the earth from all directions we could not assume its density to increase downwards in the same way as the density of the nitrogen particles which are continually formed through the condensation of atmospheric nitrogen.

The fact that the ratio of the intensity of the green line to that of the nitrogen bands is nearly independent of altitude shows, I think, that the green line must be interpreted as a nitrogen line, and taking into account the variability of the nitrogen spectrum there is nothing strange in the idea that nitrogen under the conditions present in the higher strata of the atmosphere, when bombarded with electric rays, is caused to emit lines not found under the conditions as yet experimented upon in the laboratories. As already mentioned, that part of the spectrum which by identification of lines has been found to be due to nitrogen is of a very singular type, as the number of lines in the band series are very much reduced.

Now, according to the theory of light-emission given by Bohr, the number of lines in an ordinary line series should increase when the mutual distance between the molecules increases, and the same rule may possibly hold also for band series. If then the gas in the auroral region consisted of free molecules, we should, considering the extremely small density, expect a great many lines in the series. If, however, the molecules are packed together into particles they are not free, and thus we should expect the conditions to be unfavourable for the formation of a large number of lines of the band series, and we should get a type of spectrum like that observed for the auroræ.

The Change of Colour of the Auroræ.

§ 8. A most characteristic feature of an auroral display is the sudden varied colour changes. As pointed out in previous papers these changes cannot be explained through a variation of the type of electric rays.

They are not a function of the altitude, for auroral streamers with their bottom edges at the same level may have a different colour and for the same streamer or drapery the colour may undergo sudden changes.

If the cosmic rays are penetrating into an atmosphere consisting of neutral gases, colour changes of this kind are very difficult to explain, and as a possibility I have previously pointed out that colour changes might result from a change of velocity of the electric rays produced when they pass through the atmosphere. Effects of this kind may play a part, but they cannot account for all colour changes.

If the colour changes are to be explained in this way we must assume that ordinarily a large proportion of the electric rays are turned back into space before being completely absorbed.

From laboratory experiments we know that effects of this kind occur when electric rays are retarded on their passage through nitrogen.

When the velocity is reduced the colour turns red. In the auroræ the red colour should correspond to complete absorption and the average colour should be red when a large portion of the rays are completely absorbed.

This explanation, which seems the only one possible for a gaseous state of the auroral region is, however, somewhat artificial and it meets with great difficulties. Thus we should expect that the auroral forms for which the height of the luminosity is small (Arcs) should be red because they must be formed by rays passing nearly parallel with the magnetic lines of force, and therefore they must have a great chance of being completely absorbed. But, as far as my experience goes, the arcs have always the typical greenish-yellow colour.

If the gas was electrified and if the electrification influenced the spectrum we might, as mentioned in my first communication, explain colour changes through changes of electrification of the atmosphere produced by the electric rays. As we have seen, however, a highly ionized layer of ordinary gas cannot exist, and in a dust atmosphere only a very small fraction of the molecules carry an elementary charge, and the electric charge cannot be supposed to produce

any essential change of the spectrum emitted when the particles are struck by electric rays.

The dust atmosphere, however, gives us in another way a new and simple explanation of the colour changes. To understand this we have merely to bear in mind that the typical auroral spectrum with the very prominent green line is produced when electric rays strike nitrogen dust-particles, and that a more ordinary nitrogen spectrum with the well-known prominent red bands is produced when the same rays strike isolated nitrogen molecules.

The first spectrum gives a total colour of the ordinary greenish-yellow type, the latter should produce a red average effect.

When the dust atmosphere is bombarded by very intense corpuscular radiation some particles may partly disintegrate or evaporate, and the disintegrated nitrogen will give the red colour.

According to the degree of disintegration the ratio between the intensities of the two spectral types will change, and in this way we may get a great variety of colours from green to dark red.

From this point of view we should expect that the deviations from the ordinary greenish-yellow colour would be mostly restricted to very intense auroræ with very concentrated narrow ray streamers, because these conditions are favourable for disintegration.

Diffuse and faint auroræ, however, should have the greenish colour. Thus the dust hypothesis leads to consequences in accordance with observations.

If the atmosphere was bombarded by some kind of matter which was able to penetrate into the ordinary gas atmosphere the transition should be marked by a change of colour from greenish-yellow to red. If we could observe the height where this colour change takes place we should be able to fix the lower limit of the dust layer.

Two phenomena, auroræ and meteors, might here come into consideration. If the cosmic electric rays in certain cases had a sufficient penetrating power to reach the gas layer, we should observe auroræ with a red bottom edge. I have only observed an aurora of this type on one occasion (Oct. 11th, 1912, at Bossekop). But we cannot, only from the change of colour from green to red near the bottom edge, conclude that this change marks the transition between dust and gas. To test this point we should have to take parallactic photographs and measure the height of the bottom edge. If this were of the usual height the colour change could not mark

the transition layer ; only if the bottom edge was found to be situated considerably lower than 100 km. might the change of colour possibly mark the lower limit of the dust atmosphere.

My colleague Professor Schroeter has mentioned to me that when meteors penetrate into the atmosphere and get luminous the light emitted is first greenish white, but then at a certain point of the orbit the colour suddenly turns into red. A possible interpretation of this change is that it marks the transition between the dust- and the gas-atmosphere.

The Height of the Dust Atmosphere.

§ 9. As this dust-atmosphere depends on the solar action we might expect it to be most strongly developed towards the equator. This may at any rate partly account for a very marked difference with regard to the auroræ observed near the auroral zone and those appearing on lower latitudes. As we pass away from the poles the "ray forms" seem to be more frequent and very much higher. This is also confirmed by the most interesting height measurements carried out by Störmer * at Christiania during recent years.

He finds much greater upper limits for the auroral rays at Christiania than at Bossekop, and it would seem as if the "height of the atmosphere" increased towards the equator.

We should, however, remember that there is another possibility to be taken into consideration. We know that the upper limit of an auroral streamer may vary greatly even at the same place. As shown in previous papers †, these variations are explained through the effect of the magnetic field. When the rays form a finite angle with the lines of force they will turn about them in screw orbits. When the rays come down nearly parallel to the lines of force there will be an intense light near the bottom edge, but passing upwards the luminosity rapidly diminishes and the visible light may only have an extension upwards of 15–20 kilometres.

If a considerable portion of the cosmic rays enters the atmosphere in directions nearly perpendicular to the lines of force, the number of collisions and the light intensity may be considerable very high up in the atmosphere and we get the long ray streamers.

Now it was pointed out in my previous papers ‡ that

* C. Störmer, *C. R.* t. 179, p. 109 (1923).

† *Forhandlinger ved Geofysikermøtet i Gøteborg*, Aug. 1918, p. 12 ; "The Position in Space, etc. . . . . " Geophys. Publ. vol. i. 1920, p. 159 ; Phil. Mag. xlii. p. 59 (1921).

‡ *Loc. cit.*

magnetic disturbances would increase the probability for the formation of long ray-streamers, which gave an explanation of the fact that the auroræ having long ray-streamers usually appear at the time when the magnetic storm is most intense.

Now I have also pointed out that the auroræ are carried down to lower latitudes through the effect of the perturbing magnetic fields, and thus we should expect that the chance of the formation of long streamers should be greater at lower latitudes.

In this way we should be able to explain that long rays in comparison with other forms were more frequent at lower latitudes.

It is, however, not only so that an aurora at lower latitudes has a great tendency to form long ray-streamers, but the considerable material of height measurements obtained up to the present time seems to show that the *maximum height obtainable* is greater at lower latitudes than near the auroral zone.

In the material of 434 pairs of photographs from the Haldde Observatory * no ray was found reaching a height greater than 303 km. and Størmer's † measurements from the same locality have given a similar result. At Christiania, however, Størmer ‡ finds much higher values for the upper limit. Rays reaching heights of 500 km. are not infrequent, and he has measured heights as large as 750 km.

If the distribution of matter in the auroral region were independent of the latitude we should expect that also near the auroral zone the conditions might sometimes be favourable for the formation of the very long rays. If such very long rays *never* occurred at these localities it would indicate that the height at which the density of the atmosphere falls below a certain value necessary for giving noticeable light emission should increase towards lower latitudes.

As already mentioned, such a variation of the heights of the atmosphere might be due to the variation of the solar action, but I think that also the magnetic field of the earth may act in a similar way. The electric particles will, during their upward motion, have a tendency to follow the magnetic lines of force and thus the electrified "dust" may be driven towards the plane of the magnetic equator of the earth.

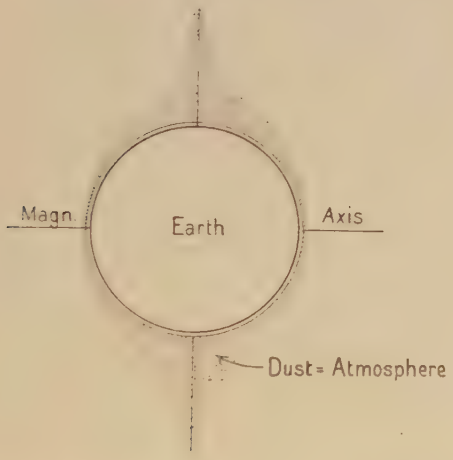
Birkeland has made a most interesting experiment which

* Geophys. Publ. vol. i. no. 1.
† Geophys. Publ. vol. i. no. 5.
‡ C. Størmer, *C. R.* t. 176, p. 109 (1923).

forms an analogy to this process. When an electric discharge is passing through a discharge tube with a magnetized sphere as a cathode, the discharge is almost entirely restricted to the region near the magnetic equator. The motion of the electrified corpuscles is on both sides directed towards the magnetic equator through the action of the magnetic lines of force. At the plane of the magnetic equator the particles will accumulate and work their way outwards perpendicular to the magnetic lines of force, driven by the electric field.

From the point of view here taken with regard to the constitution of the upper strata of the atmosphere we might

Fig. 3.



expect that the clusters would accumulate near the plane of the magnetic equator, where this dust atmosphere would reach a maximum height, and thus there might be formed a kind of dust ring round the earth as indicated in fig. 3.

In this connexion it is of interest to notice that probably currents of negative corpuscles from the sun continually circulate round the magnetic equator. When the positively charged particles are driven upwards in the equatorial regions they may meet this swarm of corpuscles and in this way get neutralised so that they again begin to fall.

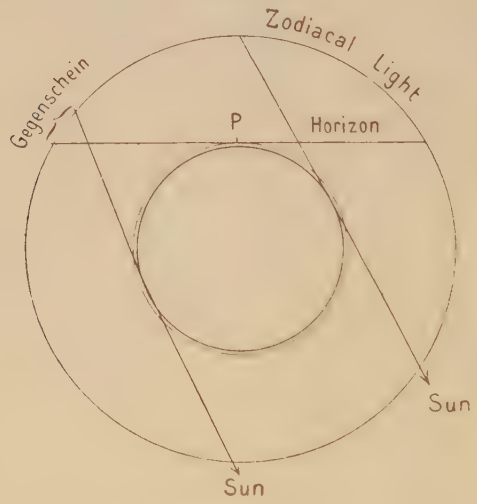
These equatorial currents of negative electricity might then act as a shield, preventing nitrogen from leaving the earth.

The Zodiacal Light, the Blue Sky, and the Twinkling of Fixed Stars.

§ 10. The dust ring described in the previous paragraph will scatter the sunlight, and such a disk of small particles is just wanted to explain the zodiacal light. Long after sunset the sun will still illuminate this dust ring, and at places near the equator we see a cone of light standing up from the horizon.

If the disk reaches to a height of the same order of magnitude as the diameter of the earth we may see the luminosity on the other side of the earth (opposite to the sun) in the form of what is called "Gegenschein." (Fig. 4.)

Fig. 4.



As this dust ring is composed of electrified particles in motion it is maintained through a continuous process, and it is to be expected that the dust ring and the zodiacal lights will not be always equally strong even at the same place and under the same light-conditions. It is even possible that fairly sudden changes may occur with regard to the intensity and outlines of the zodiacal light. Indeed, some observers, *e. g.*, von Humboldt and the Rev. George Jones, state that they have noticed very marked changes in the intensity of the zodiacal lights.

Although one of Birkeland's experiments, as we saw, is of interest as an analogy to illustrate the way in which matter is distributed in the upper strata of the atmosphere, still the

explanation here given for the zodiacal light is essentially different from that of Birkeland. According to his view, the zodiacal light should be explained in a similar way as the northlights as a direct effect of radiation from the sun, the two phenomena being produced however by different groups of rays.

The question now might naturally arise whether the ring of Saturn might not be a system of electrical dust formed in a similar way as the assumed dust-ring round the earth? If so, it would follow that Saturn must have an atmosphere and a magnetic field, but the question then arises, why have not the other planets such dust rings? Several circumstances might prevent their formation so, e. g.:

(1) The temperature may be too high for the formation of clusters.

(2) The atmosphere might consist of hydrogen and helium which might have a too low freezing-point for the formation of clusters.

(3) The planet might have no atmosphere.

(4) The planet might have too weak a magnetic field.

I shall leave these suggestions for the astronomers.

There are, however, some other points in connexion with this view with regard to the constitution of the upper strata of the atmosphere which call for considerable attention.

We have already mentioned that the dust atmosphere scatters the light. If the particles have a suitable average size the light will be selectively scattered, and the upper atmosphere, when illuminated by the sun, would seem to give out light of some colour. In this way I think we may explain the blue sky.

The blue sky then should be formed through the scattering of sunlight in the nitrogen dust layer which forms the upper strata of the atmosphere.

Simple every day experiences show that the blue sky cannot be due to scattering of sunlight in a nitrogen gas layer. The amount of gas above each unit of surface is equivalent to the quantity contained in a horizontal cylinder of unit cross-section and with a length of about 10 km. Now if it were not for the atmospheric scattering the sky should appear black in between the stars. But on a clear sunny day a black object at a distance of 10 km. appears very nearly black.

The fact that the sky is quite as blue on the top of very high mountains as near the ground shows that the great scattering resulting in the intense blue colour of the sky

must originate from a layer situated high up in the atmosphere. A continual current towards the earth of cosmic dust cannot come into consideration, for this dust cannot have a greater density high up in the atmosphere than lower down.

In the nitrogen dust, however, we have an inexhaustible source of scattering particles which can never reach below a certain height.

The density of the dust layer is very small, and the average distance between the particles comparatively great. If we drew a line through the dust atmosphere it might only cut a few particles and this number might undergo considerable and rapid changes. In this way the dust layer gives us a very simple explanation of the twinkling of the fixed stars. When we look upon a fixed star with the eye, it is the average intensity through a cylinder with a cross-section equal to the pupil of the eye which determines the observed light intensity. Inside such a narrow cylinder the number of scattering particles may undergo considerable variations, and these variations are seen as twinkling.

If, however, we look at a planet the latter has a measurable angular diameter of say 8–10″, and as the scattering layer is situated so high up in the atmosphere the cone of light reaching the eye will now, in the scattering layer, have a very considerable cross-section of say 20–80 square metres, or $4 \cdot 10^6$–$16 \cdot 10^6$ times larger than in the case of fixed stars. The variation of the number of scattering particles inside the cone relative to the total number will now be extremely small and can no longer be observed; the planets give no twinkling.

Concluding Remarks.

§ 11. If the electrically charged atmosphere were evenly distributed all round the earth it could have *no* influence on the electric field near the ground. If, however, there is an accumulation of charged particles near the magnetic equator there will be an action inwards, and the earth will be negatively charged from electrostatic induction. In this way our electrical atmosphere might to some extent influence the electric field near the surface of the earth.

The electric field of the earth will also act on the electric rays coming from the sun and modify their orbits. Negative rays will be attracted and may reach the earth further from the magnetic poles than they would have done if only magnetic forces had been acting. In this way it seems possible that even electron rays of a moderate penetrating power may come down to the earth in the auroral zone.

During strong auroral displays we often observe that the sky remains luminous for several minutes after the bombardment of the atmosphere by electric rays has ceased. And sometimes it seems as if there were secondary discharges from the part of the sky exposed to the rays. These phenomena are difficult to explain when we assume the rays to penetrate into a gas atmosphere, but if the charge is taken up by dust particles we may understand that it takes a considerable time for the charge to distribute itself.

As the ordinary gas atmosphere is bounded by a dust atmosphere of quite different mechanical and electrical properties the upper layer will reflect sound waves and may act as a shield for electric wave energy. In other words the electrified dust atmosphere forms the "Heaviside layer" round the earth.

As already mentioned, a most characteristic feature of the northlight spectrum is the great reduction in the number of lines. A few of the N-lines are very strong and a few new ones appear—but most of the lines are suppressed.

Now we know also that spectra of nebulæ have a similar character; they consist of a few lines, many of which are not found among the lines observed in laboratories. This similarity would suggest a similar constitution of the nebula and the upper strata of the atmosphere, and if our view with regard to the constitution of the auroral region is the right one it would suggest that the nebulæ consisted of fine dust particles made luminous through the bombardment of electric rays. These electric rays might be due to disintegration of radioactive matter. If the average density of the nebula is very small the temperature, in spite of some radioactivity, may be very low, just as we have supposed to be the case in the higher strata of the atmosphere.

§ 12. *Summary.*

(1) Photographs of auroral spectra corresponding to different altitudes have been obtained.

(2) The intensity of the green line relative to that of the nitrogen bands does not increase as we pass upwards, but shows a small but noticeable diminution.

(3) Our observations show that the green line cannot be due to any light elementary gas such as H, He, or the hypothetical geocoronium.

(4) The green line must originate from nitrogen or from some substance attached in some way to nitrogen. Arguments are given for considering the latter possibility to be excluded.

(5) Independent of any hypothesis with regard to the origin of the green line, observations show that nitrogen is a prominent component of the atmosphere to its very upper limits.

(6) To explain this fact we must either assume for the auroral region an increasing temperature amounting to several hundred degrees centigrade or we must assume that nitrogen is kept up by the effect of electric forces. Various circumstances seem to exclude the first possibility.

(7) The differential equation for the variation of density in an electrified atmosphere has been found, but its solution has not been carried farther because some simple calculations have shown that a highly electrified upper gas layer cannot exist.

(8) If we are sticking to the hypothesis that nitrogen is driven upwards by electric forces we are led to the assumption that the nitrogen in the upper layer is condensed into small crystal particles, which are then electrified through the photo-electric effect of sun-rays of very short wave-lengths.

(9) The auroral spectrum should be produced when electric rays from the sun penetrate into the layer of nitrogen dust.

(10) The colour changes of the auroræ are explained through disintegration or evaporation of the dust particles caused by the cosmic electric rays.

(11) Our view with regard to the physical state of the atmosphere gives a simple explanation of the following phenomena :

- (*a*) The sudden change of colour along the track of a meteor.
- (*b*) The increase of the maximum height of auroral rays as we pass towards lower latitudes.
- (*c*) The zodiacal light.
- (*d*) The blue colour of the sky.
- (*e*) The twinkling of fixed stars and the absence of twinkling in the case of planets.
- (*f*) Reflexion of electric waves and sound waves from the upper layer of the atmosphere.
- (*g*) The secondary auroral phenomena.
- (*h*) The absence of an upper layer consisting of the light gases hydrogen and helium.

Physical Institute,
University, Christiania.

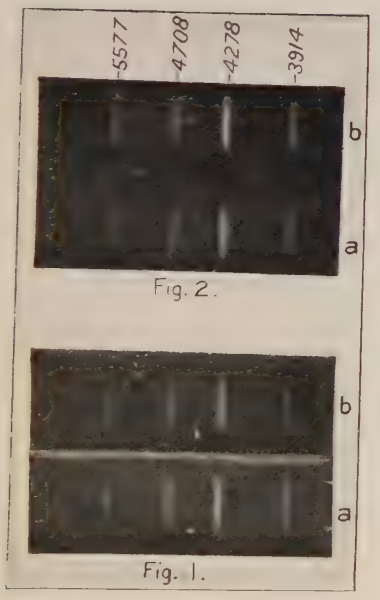
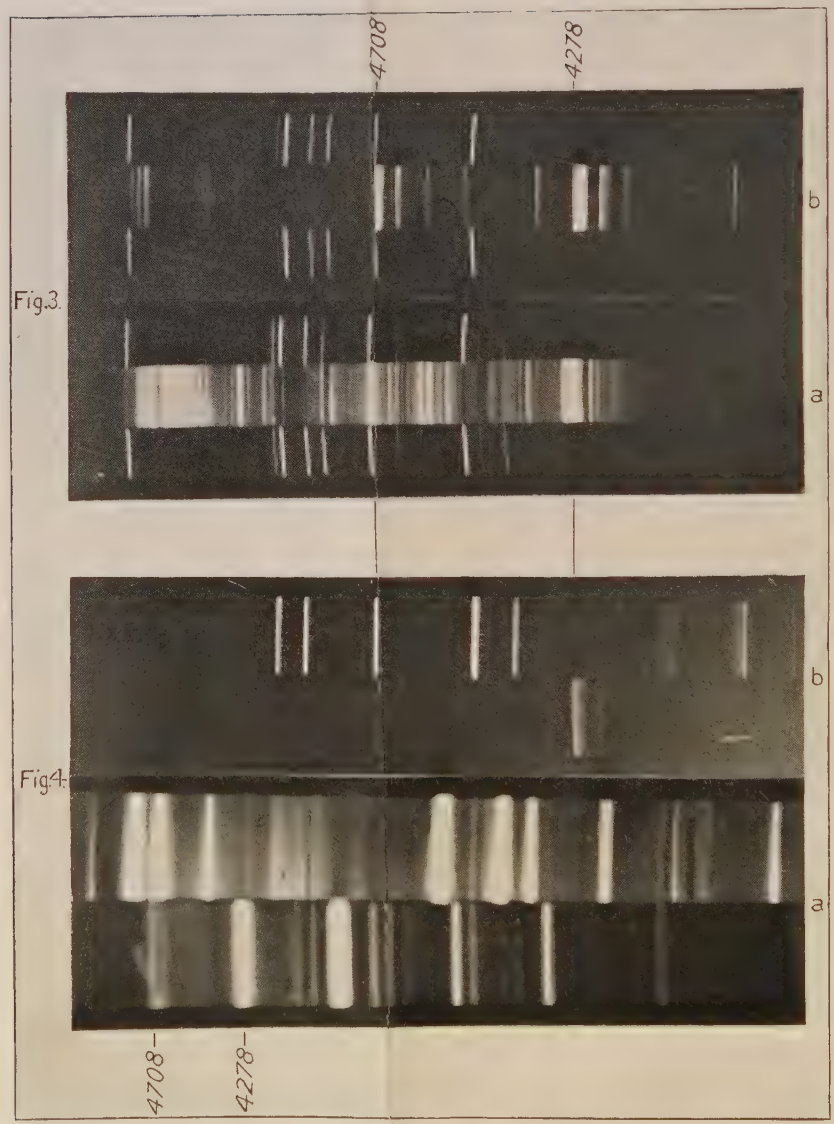


Fig. 2.

Fig. 1.


4708
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b
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Fig.3.
b
Fig.4.
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LXV. *On a Simple Method of extending the Balmer Series of Hydrogen in a Vacuum Tube.* *By* R. Whiddington, *M.A., D.Sc., Cavendish Professor of Physics in the University of Leeds* *.

IT is well known that under the usual conditions of the vacuum-tube discharge in the laboratory the hydrogen primary spectrum is limited to only a few lines, ordinarily α, β, γ, and δ †, only showing with any intensity. This, when contrasted with the fact that in stellar spectra the series is complete, appears in rather a remarkable light.

It occurred to me that in stellar spectra the conditions under which the hydrogen exists may possibly often differ from the state in a vacuum tube in, at any rate, two respects :—

(1) In being at a much lower pressure.
(2) In being bombarded continuously and intensely by electrons from the hot interior of the star.

Either of these conditions would favour the production of the primary spectrum rather than the secondary, and moreover should increase the probability of the production of those frequencies nearer the head of the series. The first condition on account of the comparative isolation of the radiating atom and the consequent realization of its possible outer orbits, and the second condition because of the greater opportunity of combination of the positive hydrogen ion with an electron rather than a negative hydrogen ion.

Unfortunately I have not been able to devise a means in the laboratory of realizing conditions (1) and (2) quite separately, but I have had some partial success.

The experiment was perfectly simple, and consisted merely in making a glass vacuum tube fitted with a hot cathode of tungsten wire. Evacuation was effected by an oil backing pump and a mercury diffusion pump, the usual very low vacua being attained with the customary ease. As was expected, when the cathode was cold the discharge refused to pass brightly with a steady stream of electrolytic hydrogen passing through when the pressure fell below a fraction of a millimetre. Photographs with the discharge brilliantly passing at a millimetre or so were taken with a large Hilger

* Communicated by the Author.

† R. W. Wood (Phil. Mag. 1921) has described a laboratory method of working which provides a much extended series.

quartz spectrograph, and showed, as usual, strong secondary spectrum and the first four members of the Balmer series with intensity rapidly diminishing from α to δ.

Increase of current up to the limit set by the generator furnishing the potential failed to change appreciably the relative intensity of these lines.

The hydrogen supply was now gradually cut down until the pressure fell to below $\frac{1}{1000}$ mm. of mercury or thereabouts. The discharge now refused to pass with the cathode cold, even with an induction coil attached to the tube, only a green glow on the glass showing. When, however, the cathode was brightly glowing, a brilliant discharge passed at this low pressure, a current of several hundred milliamperes being possible if desired.

On photographing this discharge it was found that the intensity of the Balmer series was increased relative to the secondary spectrum, and, more than that, a considerable number of Balmer lines extending into the violet appeared on the plate.

Very unfortunately the dispersion of the spectrograph near the head of the series was not great enough to follow the series very far with absolute certainty, but 19 or 20 members of the primary series were traced confidently with the help of a comparison iron arc spectrum. Further, the relative intensities of δ to γ, γ to β, and β to α were enormously increased.

How much of this extraordinary change in the intensity of the primary spectrum is due to condition (1) and how much to condition (2) I can only suggest by referring to another observation which was made when examining the tube with the cathode cold, using a small direct-vision spectroscope. I do not remember seeing any reference to this in the literature, but feel sure the phenomenon must be a familiar one to spectroscopists.

It was observed that near the cathode within the dark space the secondary spectrum was absent entirely, only the primary appearing. It is tempting to suggest that this is because the region is rich in electrons, and that the positive ions approaching the cathode have here a greater chance of picking up an electron than a negative ion, with the resulting production of primary rather than secondary spectrum.

The tube was usually supplied with potential from a direct current generator or from cells, and it was interesting to observe again the old phenomenon of regular oscillations of the current in spite of the steadiness of applied potential.

These oscillations were studied in a general manner some

20 or 30 years ago by a number of workers, and their existence has been demonstrated again in a novel manner by Dr. Appleton quite recently.

By viewing the discharge when oscillating in a specially designed high-speed rotating mirror, I have found that the oscillations often co-exist with two systems of flashes travelling through the tube with different speeds in opposite directions, but occurring at regular intervals. More particularly in argon this phenomenon is well marked, and the different colour of the flashes suggests that we have here different spectra probably due to the positive and negative ions respectively. These experiments are being followed up.

LXVI. *Note on the Ultramicrometer used as a Differential Micromanomeler.* *By* R. Whiddington, *M.A., D.Sc., Cavendish Professor of Physics, and* A. Hare, *M.Sc., Assistant Demonstrator in Physics, University of Leeds* *.

THE ultramicrometer was first described by one of us in 1920 †, and some of its applications carried out in the laboratory of this University described by Whiddington ‡ and by Sucksmith §.

The application to be noted now makes use of the same valve circuits with unimportant modifications already published. It should be added that Dowling has put on record another means for observing the change in separation of the ultramicrometer plates ‖. The heterodyne beat method, however, is considered preferable for many purposes, and whatever can be done by this method can also be done by any of the numerous possible variations of galvanometric methods.

Since the ultramanometer itself is only intended for observing changes of length in its application to the measurement of pressure, it will generally function not as an absolute, but only as a differential manometer.

Previous Instruments.

An excellent account of the various types of micromanometer so far devised is to be found in the 'Dictionary of Applied Physics,' vol. iii.

This being a readily accessible source of information, it is

* Communicated by the Authors.
† Phil Mag. Nov. 1920. ‡ Brit. Assoc. 1921.
§ Phil. Mag. Jan. 1922.
‖ Proc. Roy. Dub. 1921. Phil. Mag. July 1923.

hardly necessary to do more than mention the fact that the most sensitive manometers hitherto used measure a pressure difference of the order of 10^{-6} mm. of mercury, but are very much less sensitive when the absolute value of the pressure used exceeds 10^{-3} mm. of mercury.

The less sensitive type available for use at pressures approximately atmospheric have a sensitiveness much less—about 10^{-4} mm. of mercury.

The present arrangement, fitted up in a rather crude form, easily gave a sensitiveness much greater than that of the last-named apparatus, and exceeded detection of changes no less than 7×10^{-6} mm. of mercury at ordinary atmospheric pressures.

The Apparatus.

An ordinary aneroid barometer was taken, and in place of the pointer a small condenser plate was fitted which, as the pressure varied, altered the capacity in the ultramicrometer circuit.

In order to estimate the pressure variations corresponding to observed frequency changes, the instrument was enclosed in a large air-tight bell-jar of about 16 litres capacity, connected by a tube to a mercury column whose height could be varied by known and measurable amounts. From a knowledge of the cross-section of the mercury column and the effective volume of the bell-jar containing the apparatus, the change in internal pressure corresponding to any known change in height of the mercury column could be calculated.

In this way it was found easily possible to observe pressure changes of only 7×10^{-6} mm. of mercury. The usual electrostatic screening precautions were taken, the whole apparatus was rubber-suspended to eliminate shocks, and special oil-damping vanes were fitted on to the recording arm of the aneroid itself.

It is expected that several hundred times this sensitiveness will easily be attained by attaching the ultramicrometer to one of the more sensitive types of micromanometer as, for example, the collodion diaphragm gauge of M. Lafay.

It is clear, in fact, that the ultramicrometer is an attachment which can be fitted to any moving instrument, and performs the function of an amplifier of very great power. Further applications are in progress *.

The Physics Laboratories,
The University of Leeds.

* A highly sensitive microbalance has been under development now for nearly two years, and an experiment on the Michelson Morley lines, giving a negative result, will be noted shortly.

LXVII. *The Value of the Energy Relation in the Testing of Ferrous Metals at Varying Ranges of Stress and at Intermediate and High Temperatures.* *By* T. M. Jasper, *Assistant Professor of Engineering Materials and Engineer of Tests, Fatigue of Metals Investigation, University of Illinois* *.

IT has become desirable that more complete information should be secured for the static, elastic, and fatigue properties of metals at high temperatures. For ferrous metals a considerable number of results have been obtained on the variation of the elastic limit, yield point, and ultimate strength, but very little has been done in applying these results to metals under repeated stress at high temperatures. The application of the effect of high temperatures to repeated stress would materially assist the designer and user of steam turbines and high-speed internal-combustion engines.

In experiments on completely reversed repeated stress there seems to be a close correlation of the first order between the limit of endurance of a ferrous metal and the elastic limit, yield point, and ultimate strength when the rotating-beam method of testing is considered and if the state of the steel is comparable. In experiments on repeated stress with various ratios of reversal there is no simple correlation between the limit of endurance and the static properties of the material above mentioned.

Professor Goodman † in his tests on steels has developed a diagram showing the relation between the range of stress when repeated on a steel and the maximum stress that can be carried successfully an indefinitely large number of times. As the range of repeated stress is decreased, the maximum stress that can be carried is increased. (See fig. 1.)

The investigators working on the Fatigue of Metals at the University of Illinois are able to check the diagram of Goodman for normalized steels for a considerable range, but when heat-treated steels are considered in which the elastic-limit and yield-point values approach proportionally nearer the ultimate strength, the limits of endurance found do not follow Goodman's diagram with a good degree of correlation. Moreover, the upper limit of the range rarely goes beyond

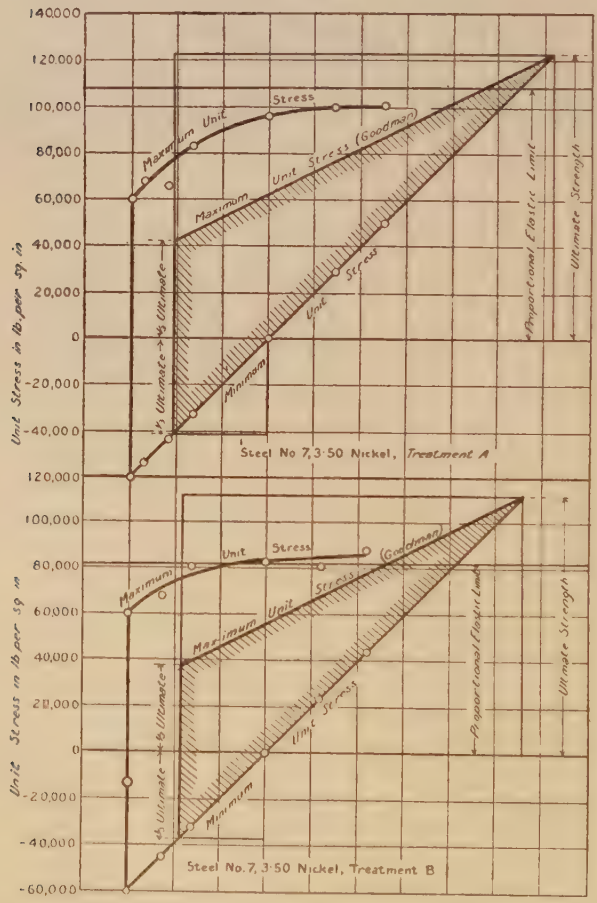
* Communicated by Dr. W. Rosenhain, F.R.S.

† J. Goodman, 'Mechanics Applied to Engineering,' Longmans, Green & Co., 1918.

the yield point of the material. This is brought out by tests on typical normalized and heat-treated steels shown in fig. 1.

Investigations of the value of the energy relation in testing have brought out the possibility of an application of this

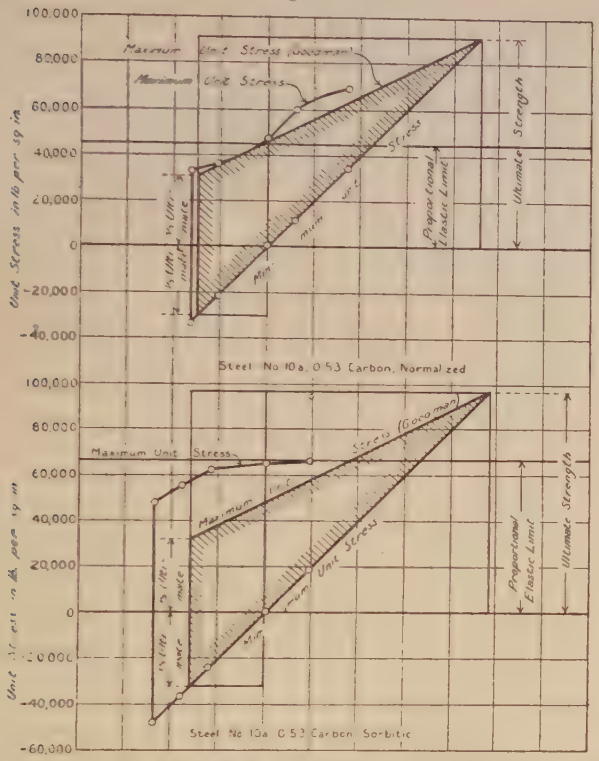
Fig. 1 *a*.



Curves showing the effect of Range of Stress on the Endurance Limit.

principle to fatigue testing at ordinary temperatures and static testing at higher temperatures, and have suggested the application of this principle to fatigue testing at higher temperatures.

Fig. 1*b*.



Curves showing the effect of Range of Stress on the Endurance Limit.

Development of the Energy Equation with Reference to Testing.

In the testing of isotropic materials the point at which the energy absorbed per unit volume becomes largest is somewhere at a surface, and is always associated with boundary conditions.

Whatever the stress system may be, it can be shown that the energy relations at a point contained within the material may be expressed by the equation*:

$$W = \frac{\lambda+2F}{2}(\epsilon_x+\epsilon_y+\epsilon_z)^2+\frac{F}{2}(\epsilon_{xy}^2+\epsilon_{yz}^2+\epsilon_{zx}^2-4\epsilon_y\epsilon_z-4\epsilon_z\epsilon_x-4\epsilon_x\epsilon_y), \quad . \quad . \quad . \quad (1)$$

where ϵ_x, ϵ_y, and ϵ_z are unit direct extensions along any x, y,

* Love, 'The Mathematical Theory of Elasticity,' Arts. 68 & 69, 1920 edition.

and z axes, ϵ_{xy}, ϵ_{yz}, and ϵ_{zx} are unit torsional extensions appropriate to these axes, λ and F are elastic constants, and W is the energy absorbed per unit volume.

In the development of the elastic theory in isotropic materials the x, y, and z axes can be so chosen that ϵ_{xy}, ϵ_{yz}, and ϵ_{zx} are zero, and for this condition equation (1) becomes :

$$W = \frac{\lambda+2F}{2}(\epsilon_x+\epsilon_y+\epsilon_z)^2 - 2F(\epsilon_x\epsilon_y+\epsilon_y\epsilon_z+\epsilon_z\epsilon_x). \quad . \quad (2)$$

The relations existing between the strain and stress are indicated by the following equation :

$$\left.\begin{aligned} \epsilon_x &= \{Sx-\rho(Sy+Sz)\}\frac{1}{E}, \\ \epsilon_y &= \{Sy-\rho(Sz+Sx)\}\frac{1}{E}, \\ \epsilon_z &= \{Sz-\rho(Sy+Sx)\}\frac{1}{E}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where Sx, Sy, and Sz are the values of the principal stresses in the principal x, y, and z directions, ρ is Poisson's ratio, and E the modulus of elasticity in direct stress for the material in question.

In static testing the conditions aimed at are to get simple tension, compression, or torsion in the surface of the material where the maximum stresses are developed. If substitutions from equation (3) are made in equation (2), and providing it is understood that with the exception of the case of a sharp re-entrant angle there are only two principal stresses possible on a boundary, and that in the ordinary method of testing in tension or compression only one principal stress need be considered, the following relations are obtained :

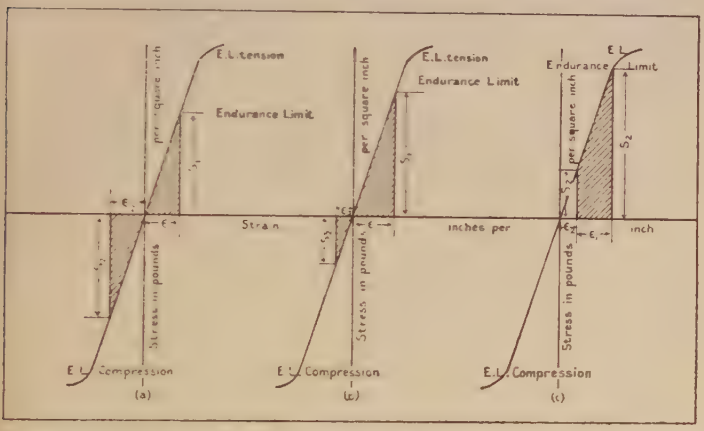
$$\left.\begin{aligned} &\text{Simple Torsion} \ldots\ldots \quad W = C_1\frac{S_{x_a}^2}{2F}, \\ &\left.\begin{matrix}\text{Simple Tension or}\\ \text{Compression} \ldots\ldots\end{matrix}\right\} W = C_2\frac{S_{x_b}^2}{2E}, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where S_{x_a} is the maximum stress in torsion and S_{x_b} is the stress in tension or compression, C_1 and C_2 are constants, the values of which are very nearly 1, and F and E are the moduli of elasticity in torsion and tension, respectively. It must be understood that the discussion applies strictly within the range where the stress is proportional to the strain.

In the testing of ferrous metals in fatigue at ordinary temperatures, it has become apparent that the energy per unit absorbed by the material at the most stressed point is

very close to a constant value. This relation applies generally to the conditions where the stress is completely or only partially reversed, provided the yield point is not exceeded. It also applies equally well, when the energy value is known, to reversed torsion, reversed direct stress, or reversed-bending results. See fig. 2.

Fig. 2.



Curves showing graphically the value of the energy relation as applied to simple tension or compression in reversed stress tests.

In the above figure,

S_1 = Stress per unit area, maximum ;

S_2 = Stress per unit area, minimum ;

ϵ_1 = Strain per unit corresponding to S_1 ;

ϵ_2 = Strain per unit corresponding to S_2.

In referring to fig. 2 (*a*) the amount of energy involved per unit for complete reversal of stress at the endurance limit is represented by the following equation :

$$W = \tfrac{1}{2}(S_1\epsilon_1 + S_2\epsilon_2), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where W is the energy involved per unit of volume per cycle of stress. The unit involved is essentially inch-pounds per cubic inch, for in equation (5) S is measured in pounds per square inch and ϵ in inches per inch. Multiplying these together gives the value in inch-pounds per cubic inch. If within the elastic limit of the material for the case of tension or compression ϵ_1 and ϵ_2 are equivalent to $\frac{S_1}{E}$ and $\frac{S_2}{E}$ respectively, where E is the modulus of elasticity, and S the

stress per unit area, and if the values of ϵ_1 and ϵ_2 thus obtained in equation (5) are substituted, the energy involved becomes

$$W = \frac{1}{2E}(S_1^2 + S_2^2). \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If the cycle does not involve a reversal of stress, the above relation becomes (see fig. 2 (*c*))

$$W = \frac{1}{2E}(S_1^2 - S_2^2), \quad . \quad . \quad . \quad . \quad . \quad (6\,a)$$

and in this case S_1 and S_2 are both values in tension or both in compression, as the case may be, or S_2 is equal to zero. For a complete reversal of stress, equation (6) becomes

$$W = \frac{S^2}{E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6\,b)$$

The term *endurance limit* has been used. For a ferrous metal, in the laboratory of the Fatigue of Metals at the University of Illinois, the endurance limit is taken to represent the maximum value of stress that a metal can withstand in repeated stress for an indefinitely large number of cycles. If a ferrous metal in repeated stress is run at a stress a very small per cent. above the endurance limit, it will usually fail before it has reached 2,000,000 cycles. If a ferrous metal is repeatedly stressed at or below the endurance limit, it will withstand at least 100,000,000 cycles of stress without failure. The time involved in experimenting beyond this point has induced the laboratory to withdraw the specimens at 100,000,000 cycles without failure*. A few experiments have been run, however, where the number of cycles have exceeded 900,000,000 at the endurance limit stress.

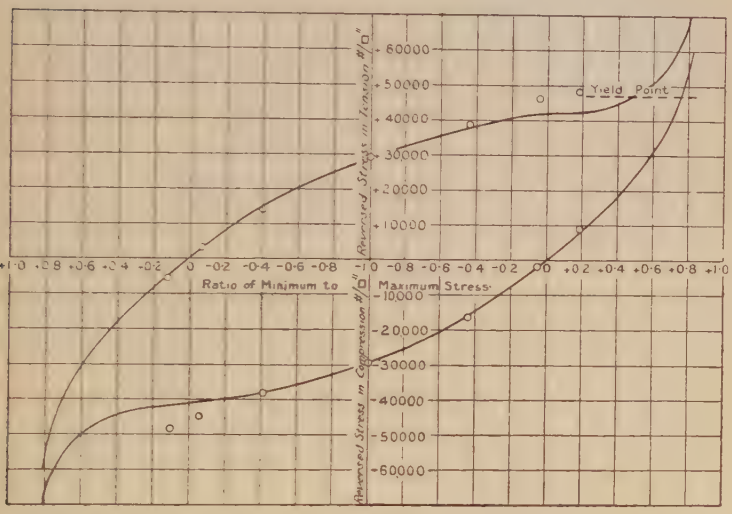
It cannot be assumed, from the nature of the stress-strain relations, that for a ferrous metal the energy constant will hold above the static yield point of the material, although in the laboratory values in some instances above this point as maximum values are obtained for endurance limits. This is explained by the fact that the yield point is probably raised by cold-working in the direction of the maximum stress in fatigue and has been verified by static tests on specimens after they have been removed unbroken from the fatigue-testing machines, the specimens having run 100,000,000 cycles without failure.

The application of equation (6) has been made to the

* H. F. Moore & T. M. Jasper, "An Investigation of the Fatigue of Metals," Series of 1923, Urbana, Ill. Eng. Exp. Sta. Bulletin No. 136.

variation of range of stress in the following manner :—The endurance limit for a complete reversal of stress has been substituted in equation (6 *b*), and the value of W established for a particular steel. With the value of the energy for any steel known, if progressive values of S_1 are assumed and the unassumed values of S_2 are calculated, a curve can be drawn which is shown in fig. 3.

Fig. 3.



Curve showing the value $W = \frac{S_1^2 + S_2^2}{2E}$ assuming that at complete reversal the value is given by $\frac{S^2}{E}$.

The plotted points are from experiments by B. P. Haigh.

This curve can be called the hypothetical curve for the material in question and for the particular type of stress involved. The experimental values of the endurance limit for partially reversed repeated stress have been plotted on this curve with the result as shown *. The coordinate along the X-axis represents the ratio of the minimum stress values divided by the maximum stress values, and the coordinate on the Y-axis represents actual stress values. This method has been adopted for endurance limits involving

* These experimental values were taken from results published by Bernard P. Haigh for Mild Steel. "The Endurance of Metals under Alternating Stresses," read before the West of Scotland Iron and Steel Institute, Nov. 1915.

torsional, direct, and bending stresses. The maximum stresses in bending with a rotating cylinder are either tension or compression, but it has been found that the rotating-beam tests for normalized or annealed steels give values about one-sixth higher than those obtained for the direct stress values when considering any particular steel. In the case of torsion, the appropriate value of the elastic constant involved is necessary if the torsional endurance limits are to be compared with the bending endurance limits. It will be noticed that the experimental values follow very closely the curve as outlined. See also fig. 1, Goodman's diagram, and the results from the laboratory of the Fatigue of Metals, University of Illinois *.

Testing Materials at Various Temperatures.

With the foregoing discussion in mind, the necessity for knowing the value of certain elastic properties of steel at varying temperatures presents itself, if the results at ordinary temperatures are to be applied to a steel at temperatures above the ordinary. This involves obtaining the appropriate values of E the modulus of elasticity and F the modulus of shear or rigidity, as it is often called, for each temperature considered.

It is to be hoped that experimental values of the endurance limits for several steels under varying temperatures may be obtained. This will mean, however, multiplying the work necessary for obtaining an endurance limit for a steel at the ordinary temperature by the number of different temperatures involved, and by a factor commensurate with the difficulty of maintaining such temperatures. The purpose of this part of the paper, therefore, is to stimulate thought on the question, to offer the results of the author's studies and experiences, to correlate some of the work done by other investigators on the question, and to apply this discussion to static experimental values at varying temperatures in which case results are already available.

The most complete work accomplished to date with reference to the variation of E the modulus of elasticity, and F the modulus of shear with varying temperatures is by Wm. Sutherland †, and for the development of the relation found by him reference is made to his publication. The following fundamental equation for rigidity has been

* F. M. Howell, Asst. Engineer, Investigation of the Fatigue of Metals, Chap. iv. "An Investigation of the Fatigue of Metals," University of Illinois Eng. Exp. Sta. Bulletin 136.

† Wm. Sutherland, "Kinetic Theory of Solids," Phil. Mag. 1891.

borrowed from Sutherland and applied identically in the manner adopted by him. To this have been added several other experimental values obtained for ferrous metals with the result as shown in fig. 4. The equation is as follows:

$$\frac{F}{F_0}=1-\left(\frac{T}{T_m}\right)^2, \quad \ldots\ldots\ldots \quad (7)$$

where

F = the modulus of shear at any temperature absolute,
F_0 = the modulus of shear at zero temperature absolute,
T = the temperature absolute at which F is obtained,
T_m = temperature absolute at which the metal melts.

Fig. 4.

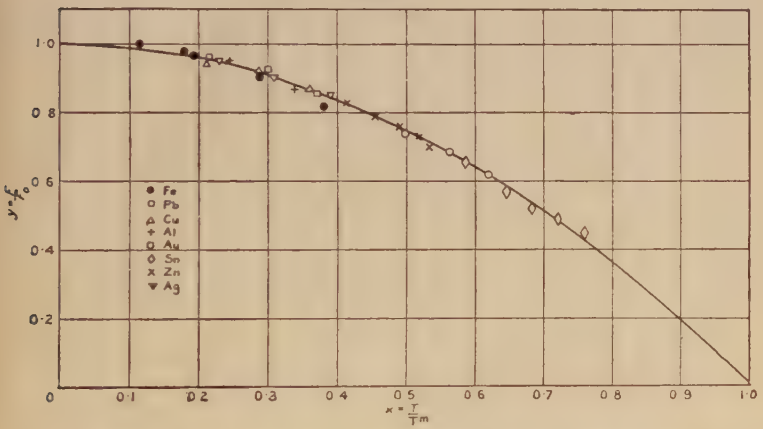


Diagram showing the Variation of the Modulus of Rigidity or Shear at various temperatures considering several different metals.
The values as shown are taken from the results published from the work of nine investigators.

$$\frac{F}{F_0}=1-\left(\frac{T}{T_m}\right)^2.$$

$$y=1-x^2.$$

Equation (7) has been used in fig. 4 in the form $Y=1-X^2$, where $Y=\frac{F}{F_0}$ and $X=\frac{T}{T_m}$. The result as shown seems to indicate very strongly that the relations for the values of the modulus of shear for varying temperatures are very well established, provided the modulus at ordinary temperatures and the melting-point of the pure metal or alloy are known.

Since the author is interested in fatigue of ferrous metals in particular, and since practically applied fatigue at high

temperatures will involve ferrous metals in the large majority of cases, the application of this relation will be carried out for ferrous metals only. It should be pointed out, however, that to apply it to any other metal requires only the appropriate substitutions in the equations and diagrams involved.

Fig. 5 shows the application of Sutherland's work to the experimental values of modulus of shear F already found for several steels. In the drawing of the curve an average value for the melting temperature of steel has been used, together with the average value obtained at the Fatigue of Metals laboratory for the modulus of shear at ordinary temperatures.

Fig. 5.

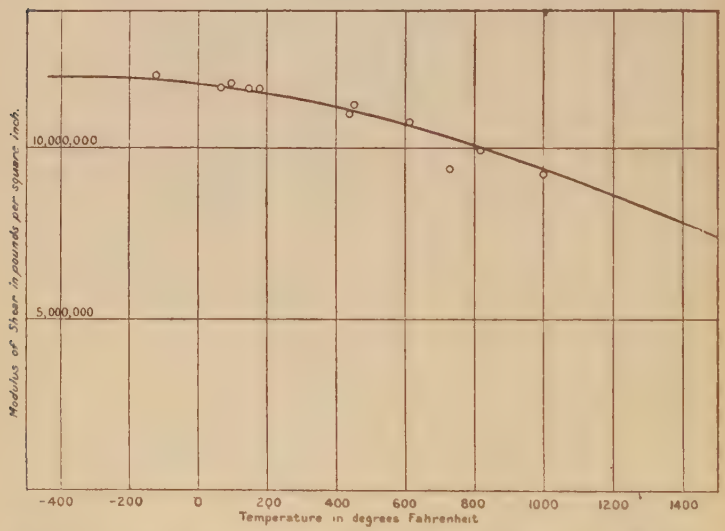


Diagram showing the variation for Steel of the Modulus of Shear or Rigidity with Temperature. The plotted values are obtained experimentally and show average results of several investigations. The results in the neighbourhood of atmospheric temperatures are the average of several determinations.

$$\frac{F}{F_0} = 1 - \left(\frac{T}{T_0}\right)^2.$$

This value of the modulus of shear is the average result of measurements of about a hundred specimens comprising about twelve different steels with a range of carbon content from 0·02 to 0·93 per cent. The remainder of the plotted values on fig. 5 have been obtained from various reliable sources.

In the case of the modulus of shear for steel it is seen that in the neighbourhood of ordinary temperatures the change due to slight temperature variation may be expected to be slight. The modulus decreases progressively, however, as the temperature is increased.

The author wishes to avoid any discussion of the Kinetic Theory of Solids as presented by Sutherland, because he is not in a position to discuss it. It is desired, however, to point out that the method adopted in presenting the application of his results gives promise of being a considerable help in the solution of a rather large problem.

The theoretical relations involved between the elastic properties of an isotropic elastic material are given in the following equation :

$$E = \frac{9BF}{F+3B} \text{ and } B = \frac{FE}{9F-3E}, \quad . \quad . \quad . \quad . \quad (8)$$

in which B represents the bulk modulus.

For the proof of equation (8) references are made to the works of Love *, Morley †, and others.

The bulk modulus B can be obtained from the kinetic theory of solids as referred to above. It is not thought necessary to reproduce the proof here, but it is given in detail in Sutherland's work ‡. The formula as applied is essentially as follows :

$$B = +\frac{2}{9}h\frac{Jc\delta}{\lambda}\left(\frac{1}{\lambda T}-4\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where

J = the mechanical equivalent of heat,
h = the specific heat of the material,
δ = the density,
λ = the coefficient of linear expansion,
c = constant for the material which is very nearly 1.
T = the temperatures absolute at which the test is to be made.

In the application of the kinetic theory of solids, Sutherland assumed average values for substitution in the above equation. It could not be expected that his experimental values would check the calculations thus found. Such coefficients as the specific heat for steel may vary one hundred per cent. between zero temperature F. and 1400° F., while the other coefficient variations by no means balance this factor.

* A. E. H. Love, 'The Mathematical Theory of Elasticity,' 1920 edition.

† Arthur Morley, 'Strength of Materials,' 1920 edition.

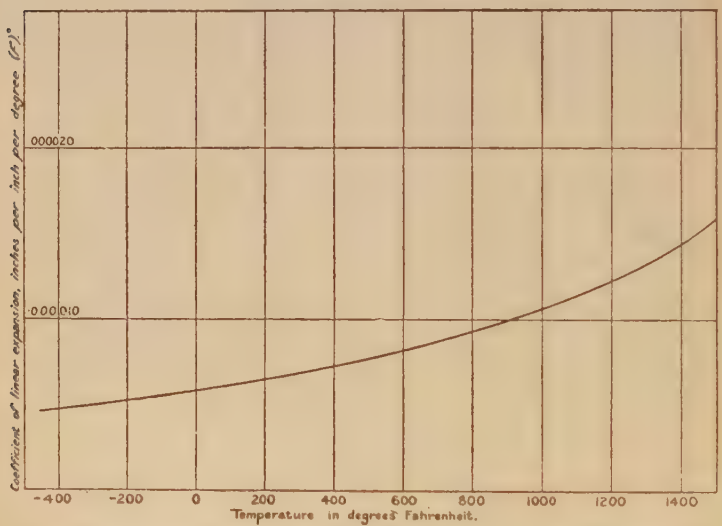
‡ "A Kinetic Theory of Solids," Phil. Mag. xxxii. pp. 524–531 (1891).

Since J and c are constants for any particular material, and δ is so near a constant that from 0° F. to 1400° F. it will vary less than 2 per cent. for steel, equation (9) will be written in the following form :

$$B = +K\frac{h}{\lambda}\left(\frac{1}{\lambda T} - 4\right), \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where K is a constant comprising all the other constant factors for steel. In fig. 6 the experimental values of the

Fig. 6.



Curve showing the Variation of the Linear Coefficient of Expansion with Temperature for Carbon Steels.

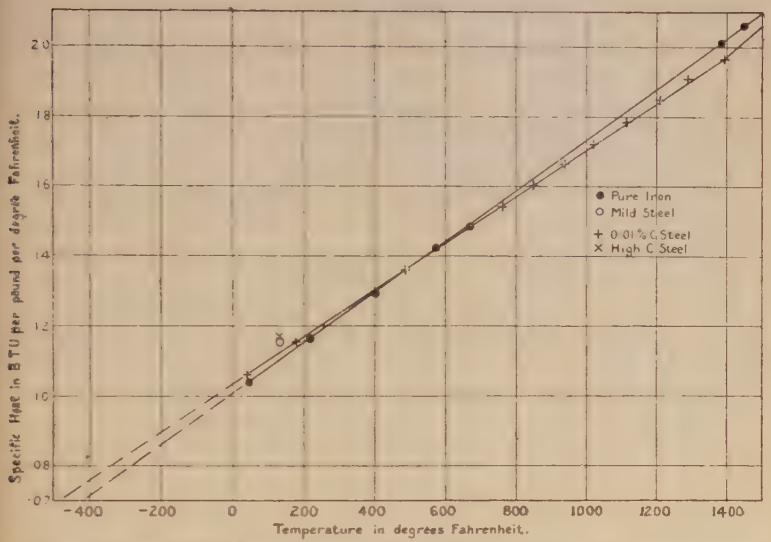
Average of seven steels.

coefficient of linear expansion for steel and in fig. 7 the specific heat of steel at various temperatures have been plotted.

From the values obtained from figs. 6 and 7 a straight substitution has been made in equation (6) for h, λ, and T at ordinary atmospheric temperature. The appropriate value of the bulk modulus as obtained from experimental results on steel from the Fatigue of Metals laboratory has been used, and from substitution in equation (10) the value of K was found to be 4·1.

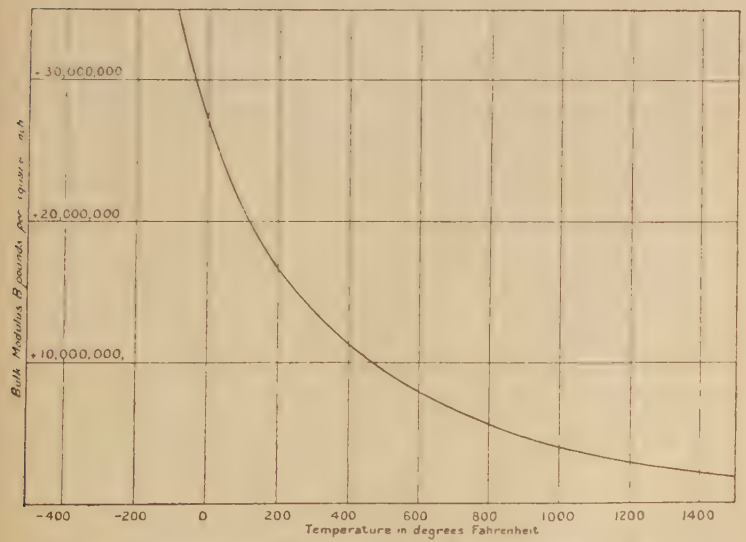
By the use of the value of K thus obtained the calculation of the bulk modulus B at various temperatures has been performed, the values being plotted in fig. 8.

Fig. 7.



Specific heat B.T.U./pound degree F. for Steel at different temperatures. Experimental values ○, ●, ×, plotted from average values +.

Fig. 8.



Curve showing the Variation of the Bulk Modulus for Steel with Varying Temperatures.

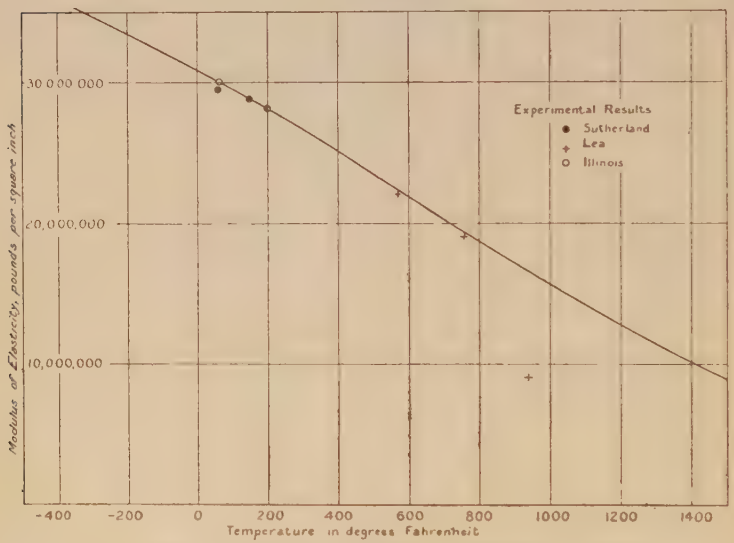
$$B = -k\frac{\hbar}{\lambda}\left(4-\frac{1}{\lambda T}\right).$$ By substitution $k=4{\cdot}10$.

Fig. 9 represents the value of the modulus of elasticity E obtained by substituting in the equation

$$E = \frac{3F}{1 + \frac{F}{3B}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8 \text{ bis})$$

the appropriate values of F and B as obtained from figs. 5 and 8.

Fig. 9.



Curve showing Variation of Modulus of Elasticity of Steel with Temperature.

$$E = \frac{3F}{1 + \frac{F}{3B}} .$$

On fig. 9 have been plotted such experimental values of the modulus of elasticity E for steel at various temperatures as the author has been able to obtain.

It might be noticed that in the neighbourhood of 950° F., the experimental value of E drops off. This suggests one of two things: either the steel in the neighbourhood of this point has an abrupt change in the modulus of elasticity due to as yet an undiscovered critical point, or there is a sharp discontinuity in the factors controlling the bulk modulus of steel at this point. That the former is the case is suggested by the plotted values of the energy relation at various temperatures shown in Table I. in the latter part of this paper. The author, however, as yet has no specific evidence on this point.

Application of the Use of the Energy Relation to Static Testing of Ferrous Materials at Various Temperatures.

If the value of the energy required to bring a steel to its yield point, considering the elastic properties involved, can be shown to be a constant, then the energy relation can be used in such a manner as to assist the materials engineer in anticipating the strength of materials at various temperatures, and also at various ratios of ranges of stress in fatigue at various temperatures.

It is to be hoped that extensive tests in fatigue at high temperatures will be made in the near future, so that applications similar to Table I. can be made in fatigue at high temperatures for several steels. Some tests on static strength of steel have already been made by H. Brearley *, and the energy involved in bringing such steel to the yield point will be investigated for the purpose of suggesting the use of the simple energy equation :

$$W=\frac{1}{2E}(S_1^2+S_2^2)=\text{constant.} \quad . \quad . \quad (6\text{ bis})$$

In the case of static testing $S_2=0$, and the value of the energy involved becomes

$$W=\frac{S_1^2}{2E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Table I. shows the application of results available on the variation of the yield point at various temperatures to the equation of energy. The elastic limit values available have not been used, because the value of the elastic limit depends in large measure on the sensitiveness of the extensometer used in making the measurements, on the initial quenching strains set up in the steel tested †, and, in the case of the variation of temperature, on the relative value of the modulus of elasticity at the various temperatures. In the tests made at the University of Illinois on the fatigue strength of metals ‡, it has been found that the yield-point values are very much more significant where they are available than the elastic-limit values, and, in the case of steels which have no yield point, that the ultimate strength is the better value to use with the endurance limit as a comparator.

* Published in Report on the Materials of Construction by the Aeronautical Research Committee, 1920, p. 37.

† Atchinson, 'Engineering Steels,' p. 114, published by Van Nostrand.

‡ H. F. Moore and T. M. Jasper, Bulletin No. 136, University of Illinois Engineering Experiment Station.

Above a temperature of 950° F., the results have not been taken as significant, because at 950° F. the experimental values of E have been found to be much lower than the theory indicates for steel. See fig. 9.

TABLE I.

Temperature, degrees F.	Yield point, tons per sq. in.	S_1, lb. per sq. in.	E. lb. per sq. in. Calculated. Fig. 9.	$\frac{S_1^2}{2E}$	Per cent. Variation from Average Value.
65	38·8	87000	30000000	126	− 3·4
212	37·0	82800	28000000	123	− 5·7
392	37·0	82800	25300000	135	+ 3·6
572	33·44	75000	22400000	126	− 3·4
752	33·6	75300	19500000	145	+11·1
842	31·32	70300	18200000	135	+ 3·6
932	28·68	64200	16750000	123	+ 5·7
1022	22·92	51400	15500000	85	
1112	20·80	46600	13600000	80	
1292	10·84	24300	12000000	25	

Summary.

From the fatigue experiments carried out by Professor B. P. Haigh at the Royal Naval College, Greenwich, and at the University of Illinois by Professor H. F. Moore and the author, it would seem that the value of the energy involved per complete cycle of stress for a specific wrought ferrous metal at the endurance limit is a constant, or if used as a constant is a safe designing factor to apply, providing the yield point of the material is not exceeded.

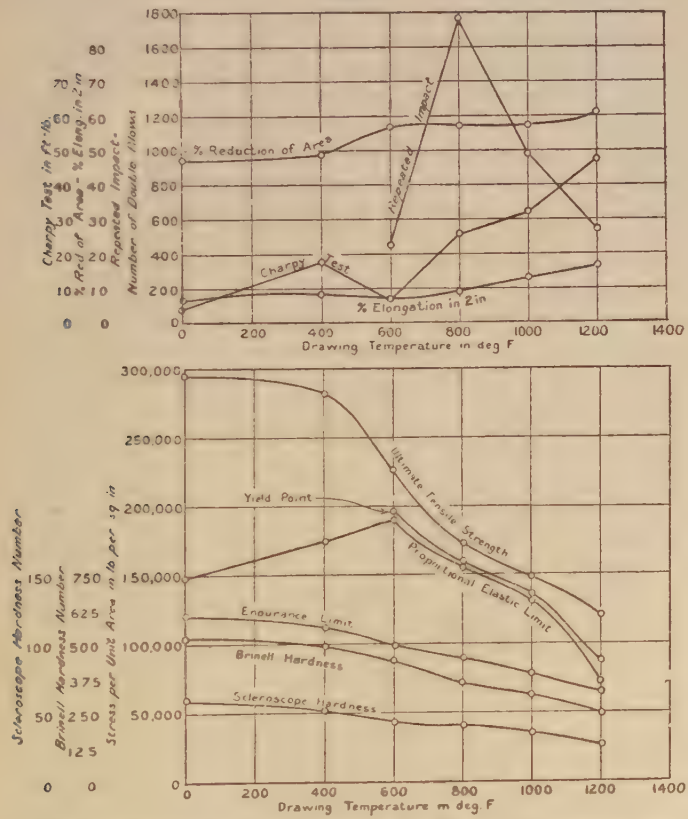
From the experiments carried out by Mr. H. Brearley, as given in the "Report on Materials of Construction" by the Aircraft Production Department of the Ministry of Munitions, it would seem that the energy involved in bringing the stress to the static yield point for a wrought ferrous metal under different temperatures is a constant at any temperature below 950° F. Above the temperature of 950° F., it seems that the value of the modulus of elasticity changes abruptly, which suggests that there is possibly a physical critical point in wrought ferrous metals in the neighbourhood of 950° F., which has not been generally recognized up to the present time.

It is felt that there is a great necessity for fatigue tests at high temperatures, and it is hoped that these may be carried out in the near future. These should be coupled

with static tests to determine the static and elastic properties for various wrought ferrous metals.

It is suggested that the constant energy idea will help in determining the safe stresses to use in internal-combustion engines and heat turbines when the endurance limits in fatigue for ordinary temperatures are determined and when the static and elastic properties at ordinary temperatures are known.

Fig. 10.


Charpy Test in Ft-lb.
Red of Area - % Elong. in 2 in
Repeated Impact - Number of Double Blows
% Reduction of Area
Repeated Impact
Charpy Test
% Elongation in 2 in
Drawing Temperature in deg F
Scleroscope Hardness Number
Brinell Hardness Number
Stress per Unit Area in lb per sq in
Ultimate Tensile Strength
Yield Point
Proportional Elastic Limit
Endurance Limit
Brinell Hardness
Scleroscope Hardness
Drawing Temperature in deg F

Curves showing the relation between the Elastic Limit and other static properties for a Strong Steel.

It is suggested that the use of the elastic limit should be made with care, because the value obtained for this property depends in large measure on the sensitiveness of the extensometer used in determining it, on the initial quenching, rolling or forging stresses set up, and on the temperature at which the material is tested. See fig. 10.

It is suggested that if, because of the particular composition or heat treatment of a ferrous metal, the yield point cannot be determined, the most consistent static property to use is the ultimate strength with an appropriate factor.

Appendix.

As an appendix to this paper it is thought wise to refer to the respective papers of McPherran *, Spooner †, Preston and Harder ‡, which deal with the effect of temperature on the static properties of steel. No values are obtainable from these papers with reference to the elastic properties of steel, although the results referred to above are of such a character as to confirm the contention of the author of this paper, namely, that the energy relations hold as a test of strength of a material at various temperatures. The greatest complication that accrues when static results of strength are obtained at high temperatures is due to the initial state of the steel.

The application of the energy relation to static tests of heat-treated steels at high temperature is undoubtedly complicated because of the fact that a quenched steel tested at ordinary temperatures is subsequently tested at higher temperatures. A quenched steel can only, strictly speaking, be compared up to the temperature of the quenching liquid which is slightly above ordinary temperature, while a steel which is quenched from, say, 1470° F., and then drawn at 1300° F., can possibly be compared in a strict sense at higher temperatures up to 1300° F. or some critical temperature which is somewhat lower. It has been found, however, for carbon steels at ordinary temperatures that, up to a drawing temperature of about 600° F., the static and fatigue properties are very little changed with the exception of possibly the proportional elastic limit. This has been considered to be due to initial quenching strains. For steels such as nickel and chrome-nickel alloys this is not, however, the case.

It would seem that it could be contended that a quenched steel tested at ordinary temperatures and of a specific carbon content differed essentially in its static properties from the same steel tested under similar conditions, and in its annealed state. So much is this the case that to all intents and purposes it might be considered as a different material, and

* "Comparative Tests of Steel at High Temperatures," A. S. T. M. Proc. 1921.

† "Discussion of Tests of Steel at High Temperatures," A. S. T. M. Proc. 1921.

‡ "Effect of Temperature on the Mechanical and Microscopic Properties of Steel," Chemical Engineer, Jan. 17, 1923.

certainly might be so classified if its chemical composition was not known.

It would seem, therefore, to be essential that the above phenomena should be considered in outlining tests of steel at high temperatures.

In reverting to the suggestion by the author that the papers referred to in this appendix tend to confirm the contention that the energy relations hold in strength of materials and are especially helpful in testing at high temperatures, it may be pointed out:

First, that the slopes of the stress-strain curves below the yield point vary progressively with the increase of temperature at which the material is tested, confirming the contention that the value of the modulus of elasticity decreases with increase of temperature. See fig. 9.

Second, that although in some cases the ultimate strength at 700° F. is higher than at ordinary temperatures, the per cent. reduction in area and the per cent. elongation in two inches are relatively decreased, and that in general a rapid increase or decrease of the ultimate strength is correlated with a decrease or increase in the other two properties mentioned.

Third, that in no case at a higher temperature is the yield-point strength greater than is found in the same steel at ordinary temperatures, and that in general the yield-point values decrease progressively as the temperature is increased.

LXVIII. *The Influence of Foreign Gases on the Secondary Spectrum of Hydrogen.* *By* S. Barratt, *B.A.* (*Balliol College, Oxford*) *.

IT has long been known that the presence of foreign gases in a hydrogen vacuum tube enhances the Balmer series relatively to the secondary spectrum of hydrogen. The effect of such gases on the secondary spectrum itself was first observed by Merton (*Proc. Roy. Soc.* 96 A. 1920, p. 382), who found that the intensity relations among its lines were greatly changed by the presence of helium in the vacuum tube. This fact is of importance in the investigation of this complex spectrum, as it isolates a comparatively small number of lines which exhibit similar intensity changes, and which are probably related. Other methods of isolating groups of lines have recently been described (Merton and Barratt, Phil. Trans. 222 A. 1922, p. 369), and in the same paper the observations on helium were extended

* Communicated by Prof. T. R. Merton, F.R.S.

over a wider spectrum range. At the conclusion of this work it appeared of interest to investigate whether gases other than helium affect the secondary spectrum in a similar way. With this object, experiments have now been made with argon and with oxygen.

For the experiments with argon a vacuum tube of the usual H type was constructed, and a palladium hydrogen regulator sealed into one arm. The tube was filled with argon at an estimated pressure of 15–20 mm. of mercury. The crude argon was taken from a cylinder, and was purified by passage over heated calcium. The tube was sealed, and hydrogen admitted through the regulator until the chief argon lines, such as λ 6032, were of about the same intensity as the secondary hydrogen lines. The spectrum from the tube was then photographed, between λλ 6500 and 4700, in the first order of a concave grating which gives a dispersion of 10 Å. per mm. A photograph of the secondary spectrum emitted by a tube filled with pure hydrogen was taken with the same setting of the instrument, and the two plates compared in juxtaposition under a travelling microscope.

It was found that argon does alter the distribution of intensity in the secondary spectrum, but that the effect is much smaller than that produced by the same pressure of helium. A list of the relatively enhanced lines is given at the end of the paper. It will be seen that most of the affected lines are also enhanced by helium, but there are several exceptions. On the other hand, many lines included in this wave-length range and markedly enhanced by helium are quite unaffected by argon. λ 6392·99 and λ 6111·08, two of the secondary hydrogen lines previously only observed in the presence of helium, proved to be developed also with argon. No lines were found to be enhanced of shorter wave-length than λ 5667, and the investigation was not extended below λ 4700, as helium itself exerts very little influence beyond this region.

The effect of oxygen was studied by an examination of the spectrum given by water vapour. A vacuum tube was prepared with a side tube containing small lumps of potash. The tube was evacuated as far as possible by a Gaede pump, and then sealed. By warming the potash, a pressure of water vapour could be liberated suitable for the purpose in view; and the potash also served as a trap for the traces of carbon gases with which a freshly prepared vacuum tube is commonly contaminated, even after careful preliminary exhaustion. In the spectrum of a tube filled with water vapour the Balmer series are strongly developed, while

the secondary hydrogen spectrum is bright enough to be photographed, though weaker than in pure hydrogen. A photograph of the spectrum was secured, as with the argon tube, and over the same wave-length range. This was compared with a plate of suitable intensity taken with pure hydrogen. Apart from a doubtful, and in any case slight, enhancement of the Fulcher bands, the secondary spectrum proved to be identical on the two plates, showing that the intensity distribution in this spectrum is unaltered by the presence of oxygen.

It must be concluded from these experiments that the great intensity changes in the secondary hydrogen spectrum produced by the presence of helium are not reproduced by other gases. Changes evidently related, though on a far smaller scale, are produced by argon, a member of the same periodic group, while oxygen exerts no influence of the kind.

TABLE of Lines in the Secondary Spectrum of Hydrogen relatively enhanced by the presence of Argon.

λ.	Effect of argon.	Effect of helium.
6495·78	+	+
6487·76	+	
6392·99	++	++
6279·85	+	
6264·89	+	
6219·40	+	
6193·80	++	+
6186·52	+	+
6181·44	+	+
6179·19	+	
6141·79	+	
6140·97 ...	++	++
6119·00	++	+
6112·80	+	+
6111·08	++	++
6085·20	++	++
6058·28	++	++
6056·10	+	++
6006·91	+	++
5998·02	+	+
5847·29	++	++
5672·73	+	++
5667·40	+	++

The wave-lengths are those given by Merton and Barratt (*loc. cit.*), and the data for the effect of helium on the lines are taken from the paper by Merton already quoted. The system of crosses indicates the degree of enhancement, a double cross indicating a greater degree than a single cross.

LXIX. *The Motion of Electrons in Carbon Monoxide, Nitrous Oxide, and Nitric Oxide.* *By* M. F. Skinker, *M.Sc., Exeter College, Oxford, and* J. V. White, *B.A., Exeter College, Oxford* *.

1. In the November number of the Philosophical Magazine an account was given of the motion of electrons in carbon dioxide. In this paper we wish to give the results of similar experiments with carbon monoxide, nitrous oxide, and nitric oxide.

The experimental part of the investigation consists principally in measuring the velocity W of the electrons in the direction of the electric force and the velocity of agitation u corresponding to various electric forces Z and gas-pressures p. The apparatus used in all these experiments was the same one which was used in the experiments with CO_2, and the method of making the measurements has already been fully described †.

2. From theoretical considerations it may be seen that the velocities of electrons u and W are functions of the ratio Z/p, and several experiments have been made with different forces and pressures in order to test these results. But when electrons tend to form ions by adhering to molecules of the gas, the proportion of the number of ions to the number of electrons increases with the pressure in an apparatus of fixed dimensions, and the values of u and W for a stream containing ions and electrons as found experimentally diminish as the pressure increases. Within the range of forces and pressures used in the determinations of the motion of electrons in carbon dioxide, and in the determinations made by Prof. Townsend and Mr. Bailey ‡ of the motion of electrons in oxygen, nitrogen, hydrogen, and argon, it was found that in nitrogen, hydrogen, argon, and carbon dioxide the velocities W and u were functions of the ratio Z/p, but in the case of oxygen the values of W and u corresponding to the larger pressures were smaller than those corresponding to the smaller pressures when the ratio Z/p was maintained constant. This effect is most marked with the smaller values of the ratio Z/p.

* Communicated by Prof. J. S. Townsend, F.R.S.
† M. F. Skinker, Phil. Mag. vol. xliv. Nov. 1922.
‡ Phil. Mag. vol. xlii. Dec. 1921, and vol. xliii. March 1922.

Thus the gases may be divided into two classes—those like nitrogen, hydrogen, argon, and carbon dioxide, where there is no tendency for the electrons to form ions and the velocities are functions of the ratio Z/p; and those like oxygen, where ions may be formed and the velocities, as found experimentally for a given value of the ratio Z/p, diminish as p increases.

We have found that in carbon monoxide, as in carbon dioxide, the electrons do not tend to form ions, but in nitric oxide ions are formed, and, as in the case of oxygen, this effect is most noticeable with the smaller values of the ratio Z/p. With nitrous oxide a different result was obtained. In this case, for the smaller values of the ratio Z/p the velocities are, within experimental error, functions of the ratio Z/p, and by a direct method of testing for the presence of ions it was found that there were very few ions in the stream. For the larger values of the ratio Z/p the velocities are not functions of this ratio, and by the direct method of testing it was found that there were a large number of ions in the stream.

3. The direct test for the presence of ions in the stream of electrons passing through a gas is made by observing the effect of the magnetic field on the charges acquired by the receiving electrodes. The values of W were calculated by deflecting the centre of the stream with a magnetic field through an angle whose tangent is $\frac{1}{16}$. If the stream consisted only of free electrons, then a magnetic field of from five to ten times as large would deflect the entire stream through this angle. This field, however, would not be sufficient to deflect particles of ionic mass by an appreciable amount. Therefore, if an undeflected portion could be detected under these conditions, it would show that ions were present in the gas.

We have found that in carbon monoxide the stream was free from ions, as it also was in carbon dioxide, nitrogen, hydrogen, and argon. With nitric oxide and nitrous oxide a small proportion of the stream could not be deflected through the small angle $\tan^{-1}\frac{1}{16}$ with a current of twenty times that used in determining W.

4. In Table I. the results of the experiments with carbon monoxide are given, and it will be noticed that there is good agreement between the values of k and W for each value of the ratio Z/p obtained with different forces and pressures. In this gas, within the ranges of forces and pressures given in the table no indication of ions was found by the direct method of testing.

TABLE I.

Carbon Monoxide—CO.

p.	Z.	Z/p.	k.	W$\times10^{-5}$.
40	4·28	·107	1·89	6·45
20	2·14		1·91	6·59
20	4·28	·214	3·04	8·80
10	2·14		3·12	8·63
20	8·56	·428	4·68	11·60
10	4·28		4·57	12·00
5	2·14		4·66	12·10
20	17·12	·856	6·36	16·00
10	8·56		6·48	16·10
5	4·28		6·37	16·20
2·5	2·14		6·48	16·60
20	34·24	1·71	10·12	21·30
10	17·12		10·08	22·00
5	8·56		9·95	22·60
2·5	4·28		10·19	22·60
1·25	2·14		10·05	22·80
10	34·24	3·42	17·47	26·30
5	17·12		17·92	26·60
2·5	8·56		17·50	26·40
1·25	4·28		17·24	26·70
5	34·24	6·85	28·30	32·00
2·5	17·12		28·25	32·00
1·25	8·56		28·45	32·20
2·5	34·24	13·70	37·20	46·40
1·25	17·12		38·70	44·80
·625	8·56		38·60	45·90
1·25	34·24	27·39	48·60	72·70
·625	17·12		47·80	71·80
·625	34·24	54·78	90·00	145·00
·3125	17·12		94·00	143·00

In Table II. the results of the experiments with nitric oxide are given, and in this case it is seen that, for each value of Z/p, the velocities diminish as Z and p increase in the same proportion. With this gas the presence of a small number of ions was detected by the direct method.

The comparison between the values of W and k for these gases and the values of nitrogen are shown by means of the curves. Figs. 3 and 4 show the values of W and k respectively plotted against values of Z/p, and in figs. 1 and 2 the first parts of these curves are plotted on a larger scale.

TABLE II.

Nitric Oxide—NO.

p.	Z.	Z/*p*.	*k*.	W×10⁻⁵.
10	4·36	·44	2·89	9·92
5	2·18		4·03	18·80
10	8·72	·87	6·12	14·5
5	4·36		6·51	21·8
2·5	2·18		6·60	29·2
10	17·44	1·74	7·44	23·4
5	8·72		8·55	30·2
2·5	4·36		9·50	32·1
10	34·88	3·49	10·60	37·6
5	17·44		11·80	42·9
2·5	8·72		12·20	42·5
1·25	4·36		12·50	42·1
5	34·88	6·98	14·85	61·4
2·5	17·44		15·40	62·6
1·25	8·72		15·70	66·8
2·5	34·88	13·95	41·6	84·9
1·25	17·44		44·1	88·8

Fig. 1.

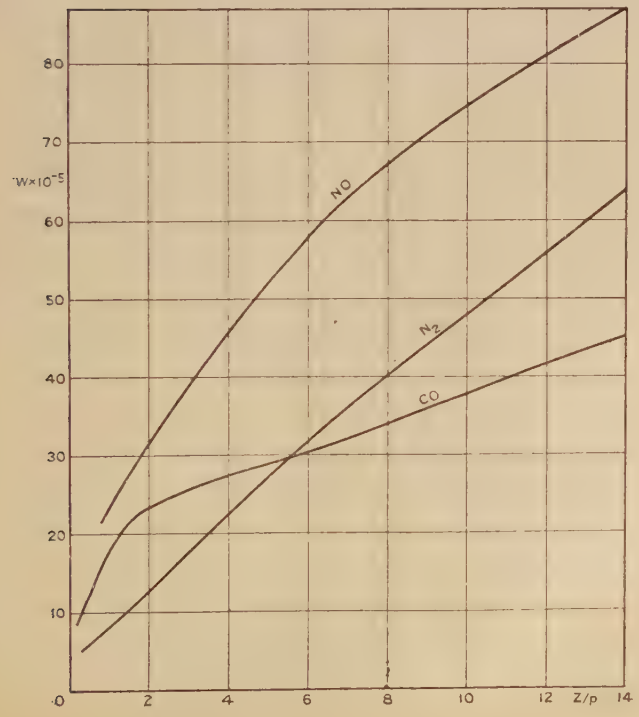


Fig. 2.

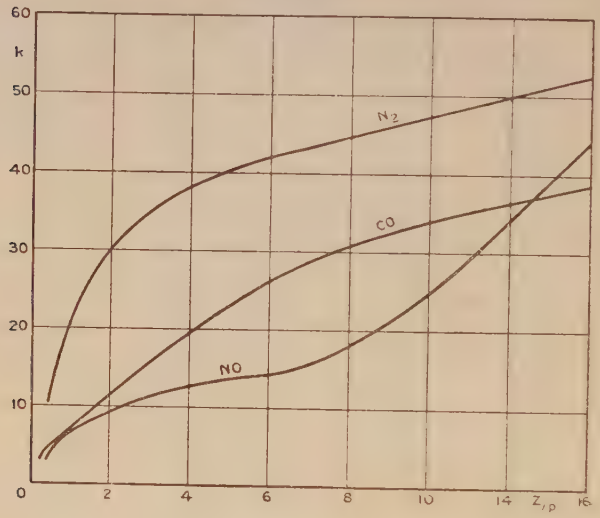
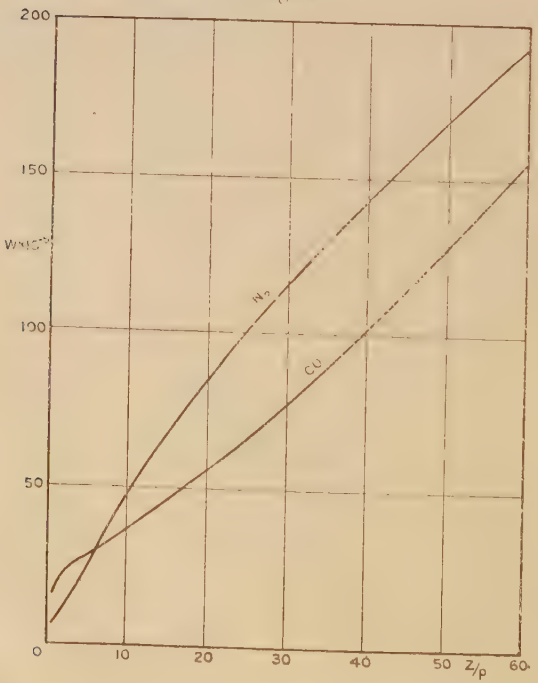
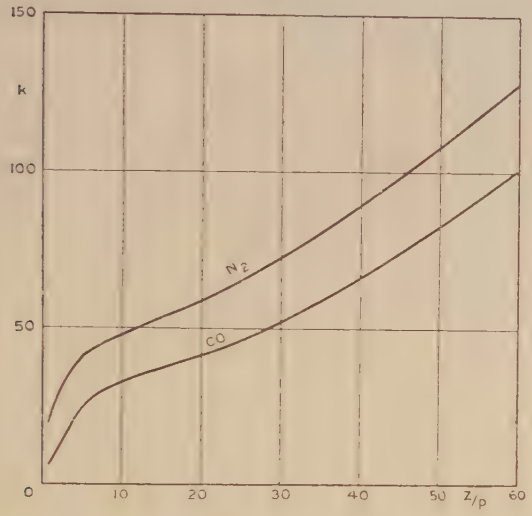

60
k
50
40
30
20
10
0
2
4
6
8
10
14
N_2
CO
NO

Fig. 3.


200
150
100
50
0
10
20
30
40
50
60
N_2
CO

Considerable difficulty was found with the nitrous oxide, as it changed slightly after it had been experimented upon, especially at the higher electric forces Z. But by using fresh quantities of gas in the apparatus consistent results were obtained. Two methods of preparation of the gas were

Fig. 4.



used, and identical results were obtained. In the first section of Table III. the agreement of k and W is good for the same values of the ratio Z/p, but in the second section there are a great many ions in the stream, probably due to the decomposition of the gas under these conditions.

5. The values of the mean free path l of the electrons in centimetres, the effective radius σ of the molecules of the gas in centimetres, and the proportion λ of the energy lost in collision with a molecule are given by Tables IV. and V. for carbon monoxide and nitric oxide.

The quantities σ_1 at the top of the tables are the values of the radii obtained by the viscosity method *.

In the values of λ for the carbon monoxide there is a maximum when k is equal to 5·5. Also in the carbon dioxide there was a similar maximum when k was equal to 20.

* Jeans, 'Dynamical Theory of Gases,' p. 327 (1921).

TABLE III.

Nitrous Oxide—N_2O.

	p.	Z.	Z/p.	k.	W$\times10^{-5}$.
1 ...	10	4·17	·42	1·32	7·31
	5	2·08		1·32	7·66
	10	8·33	·83	—	16·10
	5	4·17		1·81	16·40
	5	8·33	1·66	2·69	32
	2·5	4·17		2·71	32
2 ...	5	16·67	3·33	4·65	42·5
	2·5	8·33		4·99	49
	1·25	4·17		5·28	48
	2·5	16·67	6·66	8·05	46
	1·25	8 33		9·07	50
	·625	4·17		10·50	50
	1·25	16·67	13·33	12·60	46·5
	·625	8·33		15 15	49

TABLE IV.

Carbon Monoxide—CO.

$\sigma_1 = 1{\cdot}90\times10^{-8}$.

Z/p.	k.	W$\times10^{-5}$.	$u\times10^{-6}$.	$lp\times100$.	$\sigma\times10^{8}$.	$\lambda\times10^{4}$.
50	86	152	106·7	2·24	2·02	499·0
40	68	114	94·6	1·87	2·22	357·0
30	52	80	83·0	1·53	2·45	228·5
20	42	57	74·5	1·47	2·50	144·0
10	34	38	67·0	1·76	2·29	79·1
5	22·7	28·5	54·8	2·16	2·06	52·8
3	16	26	46·0	2·76	1·83	78·6
2	11	23	38·2	3·04	1·74	89·1
1	5·5	18	27·0	3·36	1·66	109·3
·5	4·8	13	25·2	4·52	1·43	65·5
·25	3·2	9	20·6	5·14	1·34	46·9

The energy of agitation of the electrons is proportional to k, and may be expressed in terms of the potential required to impart an equal quantity of energy to a particle having an atomic charge. This potential in volts is $k/27$. Thus

electrons lose a large proportion of their energy when they collide with molecules of carbon monoxide with velocities corresponding to ·2 volt, and of carbon dioxide with velocities of ·74 volt.

TABLE V.

Nitric Oxide—NO.

$\sigma_1 = 1{\cdot}88 \times 10^{-8}$.

Z/p.	k.	W$\times 10^{-5}$.	$u\times 10^{-6}$.	$lp\times 100$.	$\sigma\times 10^{8}$.	$\lambda\times 10^{4}$.
13	37·0	38·0	70·0	1·42	2·55	74·3
10	21·7	23·6	53·6	·88	3·24	47·7
8	16·7	18·0	47·0	·73	3·54	36·1
5	13·3	14·5	41·9	·84	3·31	29·4
4	12·2	13·9	40·2	·97	3·10	29·5
3	11·0	12·5	38·2	1·10	2·89	26 4
2	9·3	10·0	35·1	1·21	2·75	20·0
1	7·0	6·5	30·4	1·37	2·59	11·2

Note.—In a previous paper by Prof. Townsend and Mr. Bailey * some cases were mentioned where there is a comparatively large loss of energy when electrons collide with molecules, and it was implied that in all such cases there are ions in the gas—for example, in carbon dioxide. The statement was based upon the fact that k was found to be small in some of the earlier experiments with this gas, and the value of k for ions must be approximately unity. Large values of k can only be obtained with electrons, but the recent experiments with carbon dioxide †, where both k and W were measured, show that k may be very small for a stream of electrons. In this gas the velocity W is very large, even when Z/p is small, due to the fact that the electrons lose a large proportion of their energy in collision, but do not form ions.

* Phil. Mag. vol. xlii. Dec. 1921, p. 891.
† M. F. Skinker, Phil. Mag. vol. xliv. Nov. 1922.

LXX. *On Types of Crystal Symmetry in which no finite symmetrical crystal unit having the complete symmetry of the crystal is to be found.* By WILLIAM BARLOW, *F.R.S.* *

THE geometrical study of crystal structure is based on the law of rational indices as exemplified by the variety of forms of crystal symmetry presented in nature. Some 30 years ago, 230 distinct types of homogeneous structure were discriminated according to one or other of which alone the parts of a given individual homogeneous crystal must be regarded as arranged †.

Every similar unsymmetrical space-unit of the space-filling uniform partitioning of a given individual crystal is by every coincidence operation characteristic of the particular type of the 230 concerned transferred to an entirely distinct position ; the original and derived positions show no overlapping whatever.

For the purpose of visualizing this condition of things for some of the more regular types of the 230, the writer constructed models in which similar dolls' hands were placed in an open framework in relative positions located by the characteristic coincidence operations of one or other of these types ‡.

In order to derive some partitioning of space appropriate to an individual case of any given type, every doll's hand in the appropriate artificial structure can be supposed to grow similarly, and to cease growing wherever and so soon as the moving boundaries come in contact with one another. Space is in this way conceived to be partitioned without leaving interstices into similar grotesque cells presenting in nearly all cases more than one orientation. All of these cells are capable of being brought to coincidence one with another by some appropriate coincidence movement or mirror-image operation characteristic of the type of the 230 in question. In cases of types presenting a mirror-image relationship of some kind, two sets of dolls' hands are employed, rights and lefts equally numerous and similarly distributed.

It is obvious that the space-unit of this conception is destitute of symmetry.

* Communicated by the Author.

† Groth's *Zeitschr. für Kryst.* xxiii. p. 1, xxv. p. 86. "Report on the Structure of Crystals," Brit. Assoc. Glasgow, 1901.

‡ He exhibited these models at a Royal Society soirée and elsewhere in this country ; also on the Continent. They are now in the Natural History Museum at South Kensington.

The important question arises:—Is there any evidence of the universal presence of units of any nature, space-units or other, which epitomize the crystal symmetry?

It is clear that there is no geometrical necessity for the presence of such a representative symmetrical unit; the crystal symmetry is completely developed by the characteristic symmetrical operations of the type of crystal symmetry concerned as applied to a quite unsymmetrical unit. It may, however, be asked:—Is it not possible that while the space allotted to the crystal unit is unsymmetrical, this unit somehow presents or constitutes part of a form which is an epitome of the crystal symmetry concerned?

There is a serious objection to this suggestion. No crystal symmetries are known which are not comprised in those of the 230 types referred to. Other symmetries equally high but quite distinct from these are found in organic nature—*e. g.*, the five-fold symmetry occurring in some vegetable and animal structures. Why does no crystal present a symmetry outside the 230 types; for example, one corresponding to the completely symmetrical pentagonal dodecahedron? If the crystal unit is in all cases a perfect epitome of the crystal structure, what essential limitation is it which excludes this and all other highly-symmetrical types of symmetry which are not such as are completely developed by the coincidence operations of one or other of the 230 types?

Again: If, for argument's sake, the possibility of the unit in all cases possessing a symmetry which is an epitome of the crystal symmetry is accepted, there is another fundamental difficulty.

A large majority of the 230 types admitted to be geometrically possible for crystals will in this event owe their symmetry entirely to the form of the unit and not even partially to the relative situation of two or more units. This will apply to all types in which any one of the minimum group of coincidence operations is a screw-movement.

For the relation between the individual space-units of an infinite linear series, the presence of which series is indicated by the existence of a screw-movement connecting the units, is in no case consistent with any aggregation of units thus connected being finite; a selection of two or more units to form a finite group is quite arbitrary. Nearly three-fourths of the 230 types are affected.

A word as to the residue of the 230 types in which the existence of a composite finite group formed of several units is not precluded.

These composite similar groups will be regarded as 1

symmetrically-constituted aggregates of units, which latter, taken alone, may be either unsymmetrical or possessed of a symmetry equal to or greater than that presented by the crystalline mass. This is manifestly in harmony with the view that in the majority of cases the homogeneous crystal structure owes its symmetry to the symmetrical disposition, and not to the symmetrical form of identical units of which it is composed.

The author may remark that owing to some misconception his previous work has been cited as supporting the idea that every crystal is built up by the repetition throughout its volume of a certain unit which as to its symmetry is a perfect epitome of that of the whole *. A perusal of his contributions to the 'Mineralogical Magazine' and to Groth's *Zeitschrift*, published in the 'nineties, will show that then, as now, he held the contrary to be proved.

LXXI. *A Note on Dr. Hartridge's Paper on Visual Acuity.* *By Instructor-Commander* T. Y. Baker, *R.N., Admiralty Research Laboratory, Teddington* †.

IN a paper by Dr. H. Hartridge on "Physiological Limits to the Accuracy of Visual Observation and Measurement" (Phil. Mag. No. 271, July 1923) Section IV. is devoted to Interpolation Methods of Measurement. The section deals with the ability of the eye to estimate the proportionate position of a datum mark between two successive divisions of a scale, and the conclusions reached by the author are that there is considerable inaccuracy in making this type of measurement. His conclusions seem to be based upon an attempt to subdivide a millimetre into tenths when placed at a distance of 25 cm. from the eye.

It appears to me that a single test of this description is inadequate, and that Dr. Hartridge has not given the eye sufficient credit for its power to make an accurate interpolation. He has dealt with one case only—a case in which the division to be subdivided subtends an angle of less than a quarter of a degree at the eye. Without question the magnitude of that subtended angle is of importance. If it is too small and if it is too large, accuracy of interpolation is bound to suffer, and there must be some optimum angle for which interpolation acuity reaches its maximum. Some time

* Transactions Chemical Society, vol. cxxi. p. 2766 (1922).

† Communicated by the Author.

ago I made some rough experiments on this point, but unfortunately the results were not published, and I am unable at the present moment to lay my hands upon them, but to the best of my recollection acuity is greatest when the division subtends at the eye an angle of about three degrees. If Dr. Hartridge had carried out his test as an attempt to subdivide a centimetre placed at the near point of vision, I think he would have found that the eye would make a much better showing than his Table III. exhibits. For example, in his attempt to place the datum mark one-tenth of the distance from one end his error is so great that the position is very nearly one-fifth. Errors of a similar character, but less pronounced in value, occur at other places.

As against these poor results, I found that, provided the whole division appears of suitable size to the eye—and in an actual measuring instrument this can always be secured by a reading microscope of adequate power,—a moderately skilled observer will always read by estimation the position of the datum mark accurately to one-fiftieth of a division. A highly-skilled observer will do better than this, and I understand that at the National Physical Laboratory there are a number of persons who habitually subdivide by estimation accurately to a one-hundredth part. Reading and setting are not quite the same thing, and possibly not quite so high an acuity can be reached with the latter operation as with the former.

A deduction can be made from the above facts that is of great importance in the proper design of instruments of measurement. It is that it is unnecessary to employ a vernier if it is not required to subdivide a scale-division below one-fiftieth part. The eye can estimate with sufficient accuracy, provided a suitable optical magnification is fitted so as to make the angular value of the scale-division at the eye about three degrees. Yet how commonly does one see a vernier fitted for the purpose of subdivision very much coarser than this. Verniers to read twentieths or tenths are frequently found. I have even seen a barometer that had a vernier to read fifths!

It may also be of interest to readers of Dr. Hartridge's paper to add a short note on the acuity of the eye on separating two bright points.

I have recently had under construction at the Admiralty Research Laboratory a prismatic astrolabe, to be used for determination of latitude and longitude in geodetic survey.

The instrument is used by noting the instant of contact in the field of view of a telescope of two images of a star, one of which moves downwards and the other upwards. The acuity of the eye is rather better in noticing this instant of contact than in resolving a double star, but is not so good as we required. We got over the difficulty by placing across a part of the light that went to form one star image a small refracting prism, and thereby produced a double image in place of the single one. The prism was so arranged that the duplicate images were in a horizontal line, and had an angular separation at the eye of about half a degree. The instant of contact was then noted by observing when the single image moving downwards passed between and was in line with the double image moving upwards. A laboratory trial of about five hundred observations gave a mean error at the eye of about 12″ for one observer and 18″ for another. These errors are only about one-third as great as when two single images are used.

LXXII. *Radio-active Disintegration Series and the Relation of Actinium to Uranium* *. *By* A. S. Russell, *M.A., D.Sc., Student and Tutor of Christ Church, Oxford* †.

§ 1. *Introduction.*

THE object of this paper is to inquire how far existing data relating to the members of the three known disintegration series throw light on the subject as a whole, and particularly on the relation of the actinium to the uranium series. The writer has been able to deduce that there are probably four independent disintegration series, and to give the path of disintegration of each of them; to conclude that actinium arises probably from an isotope of uranium of atomic weight 239; to establish rough empirical relations between the periods of known radio-elements and to predict those of undiscovered ones, and to suggest the possibility of new end-products. He is aware of the difficulty of generalizing or in making deductions from the few facts there are relating to the disintegration series. The success of the Geiger-Nuttall relation, however, and of the Group-Displacement law, and the help afforded by empirical

* A preliminary account of part of this work appeared in 'Nature,' vol. cxi. p. 703 (1923).

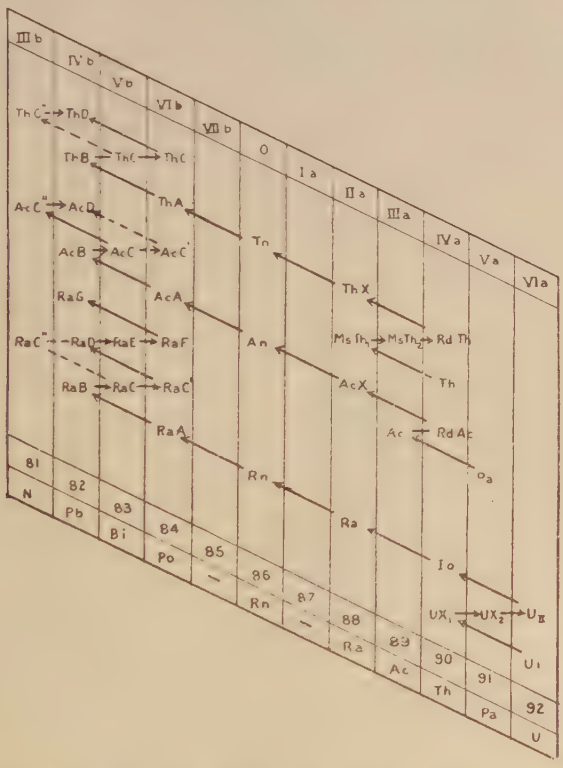
† Communicated by the Author.

relations like the α-ray and β-ray rules of Fajans have shown that deductions of this class are not without their uses.

§2. *Deductions from Analogies in the three Disintegration Series.*

In fig. 1 are set out the three disintegration series in the usual form, α-changes being represented by the upwards-sloping arrows. β-changes by horizontal arrows. The nomenclature is that of the International Table for 1923.

Fig. 1.



The figure omits the branch-products of uranium X_1 and the bodies which precede proto-actinium in its series, but, otherwise, it represents the whole of the radio-elements known with certainty at the present time.

The following facts, obvious from the diagram, will be made use of in what follows, and will for convenience be

referred to as Rules (z is used for atomic number, a for atomic weight) :—

I. A series commences with the expulsion of an α-particle followed by one or two β-particles, and does not end immediately after a succession of α-particles.

II. No bodies of atomic numbers 93, 87, or 85 appear to exist. Corollaries of this are that bodies of $z=92$ and $z=84$ expel α-particles only, and those of $z=89$ β-particles only.

III. There are only three types of successive disintegration at present known :

(*a*) A succession of at least 4 α-particles without the expulsion of separate β-particles. This will be called a $\Sigma\alpha$ change.

(*b*) An α-particle followed by two β-particles in succession and then an α-particle. This will be called a β_1 change.

(*c*) An α-particle followed successively by a β-, an α-, and a β-particle, or a β-particle followed successively by an α-, a β-, and an α-particle. Either of these will be called a β_2 change.

A change like that of RaA to RaG consisting of two β_1 changes in succession will be called a $\beta_1\beta_1$ change; a β_1 change followed by a β_2, *i.e.*, in succession an α-, β-, β-, α-, β-, α-, β-particle will be called a $\beta_1\beta_2$ change.

The following is deduced from a consideration of the atomic weights of radio-active isotopes on any accepted theory of the origin of the actinium series :—

IV. The maximum difference between the atomic weights of members of any group of isotopes is 8.

The following is deduced from a consideration of the half-value periods of radio-elements :—

V. In β_1 and in β_2 changes the period of the first β-particle body is greater than that of the second.

Finally, to simplify the discussion the following limiting assumptions are made :—

VI. (*a*) The atomic weights of isotopes obey Aston's whole-number rule ; (*b*) When branching occurs interest is confined to the main series; (*c*) α- and β-particles are the only particles expelled in a radio-active change.

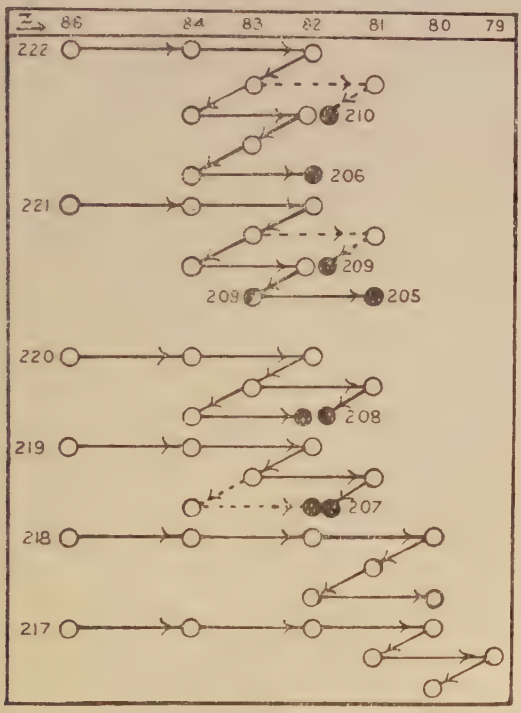
§3. *Possible Independent Disintegration Series.*

For convenience the disintegration series will be named after the number equal to the atomic weight of the $z=86$ member, 222 and 220 representing the uranium and thorium series respectively. The sequence of changes occurring in

these and other hypothetical series *after* the emanation will first be considered, and later those occurring *before* the emanation.

(*a*) *Changes after the Emanation.*—These are shown in fig. 2 for series 222, 221, 220, 219, 218, and 217, the paths of disintegration of series 221, 219, 218, and 217 being deduced from those of 222 and 220 by symmetry. Other series have not been included because their existence is

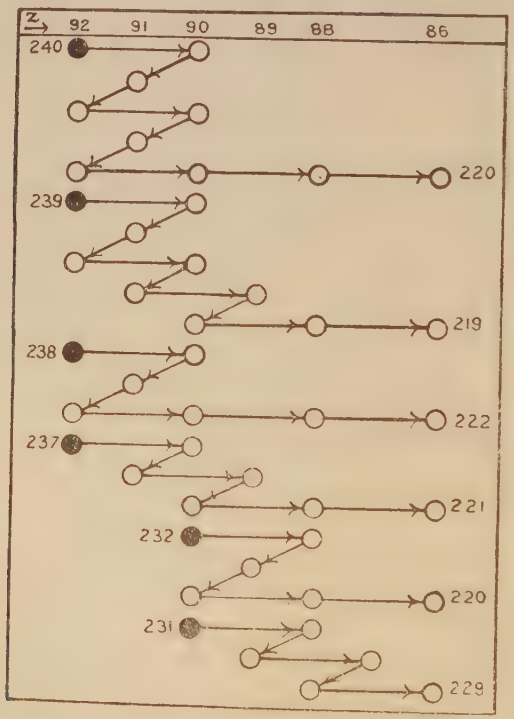
Fig. 2.



rendered improbable from the statements of §2. It is not likely that series 223, 224, etc., exist. If such did, their $z=84$ members would have atomic weights $a=219$, 220, etc., which exceed by more than 8 units the atomic weight of polonium ($z=84$, $a=210$). These series might, however, avoid passing to $z=84$ directly, *i.e.*, the $z=86$ member might emit β-particles, but at present this appears unlikely. There is no such objection to the hypothetical series 221 and 219. They should pass from their $z=84$ to their $z=82$ members by changes $\beta_1\beta_2$ and β_2 respectively, as shown in fig. 2. Series 217 is not unlikely, but 218 is. The latter

would be expected by analogy to pass from its $z=82$ to its $z=80$ member by a β_1 change, the former by a β_2 change. As the isotopes of $z=79$, 80, 81, and 82 are inactive isotopes it is reasonable to expect that such changes might occur. One of the $z=80$ members of series 218, however, would have an atomic weight of 206, which by Rule IV. renders its existence improbable since the lightest isotope of mercury, according to Aston, has $a=197$. There is no objection to series 217 on this ground.

Fig. 3.



(*b*) *Changes before the Emanation.*—Suppose next that a similar set of changes, as are assumed to occur after the emanation members of series, occur at the beginning. These are shown in fig. 3. The atomic weights of the first members of the 3rd and 5th of these are known (being 238 for uranium I and 232 for thorium) ; in consequence those of the first members of the series as arranged are in order 240, 239, 238, 237, 232, and 231. Rule IV. renders improbable

the existence of series that start on atomic weights greater than 240 or less than 231. It follows that possible series are limited to those shown, and are as numbered in the figure.

It is seen that there are two possible 220 series and two 219. The first 220 series is improbable because it would contain a body $z=90$, $a=236$, which has an atomic weight 9 units greater than $z=90$, $a=227$, which will later be shown to be radio-actinium. The second 219 series is improbable because its β_2 change contravenes Rule II. Series of lower number than 219 appear improbable because their existence would contravene either Rule II. or Rule IV.

It thus appears likely from these simple analogies and reasonable assumptions that there are only four complete radio-active series of the type that is known, namely 222, 221, 220, and 219. The path of disintegration of each of these appears from the analysis to be unique. 221 is a new series which will be sought for. Branching in the upper parts of the series is, however, not thereby excluded, because the branchings known to occur in the lower parts have not been considered. This point and empirical relations connecting branching factors will be discussed in a subsequent paper.

§4. *The Relation of Actinium to Uranium.*

In this section it will be shown that series 219 is the actinium series. The lower part of 219 is obviously that of the actinium series, and this fact, with the assumption that *the type of β-ray change is a simple function of the atomic weight*, establishes the point. Another argument in favour of this will be given later, but before that is done a short account of the rival views with respect to the origin of actinium will be given.

Boltwood * was the first to establish quantitatively a connexion between the actinium and uranium series by showing (*a*) that in old, unweathered uranium minerals the ratio of the activities of uranium and actinium was constant, and (*b*) that no actinium was present in thorium minerals. This result was at first interpreted as proving that the actinium series necessarily arose as a branch series, and the branching was believed to occur most probably either at uranium II or uranium I. This is still the most favoured view †. It is thought that the dual change occurs at uranium II, about 97 per cent. of atoms forming ionium and the radium series,

* B. B. Boltwood, Phys. Review, vol. xxii. p. 320 (1906).

† It is that given in the works on Radio-activity of Rutherford, Soddy, Meyer & Hess, Fajans, Russell, v. Hevesy & Paneth.

and the remainder forming uranium Y, the product of which, proto-actinium, is the parent of the actinium series. On this view the atomic weight of actinium is 226.

Fajans, from a general consideration of his α-ray and β-ray rule, first suggested the possibility of actinium's atomic weight being 227. In 1917 Piccard *, from a consideration of the Geiger-Nuttall relation, put forward the view that possibly the actinium series may be a distinct primary one arising from a hypothetical isotope of uranium of $a=240$. This view was criticized † as being consistent only if this isotope has the same period as uranium I—an unlikely supposition. Adams ‡ showed, however, that this proviso was unnecessary by explaining how the ratio of the ultimate parent of actinium to uranium I might be the same everywhere on the earth at any time although it would change progressively at a rate dependent on the difference in period of the two isotopes. Adams showed also that the end-product of the actinium series should by analogy with those of the other series have an atomic weight of 207. He assumes that a body $z=91$, $a=235$ produces actinuranium ($z=92$, $a=235$), which disintegrates to proto-actinium through uranium Y. On this view the atomic weight of actinium is 227. Recently Piccard § has put forward a more detailed view with regard to the independent origin of actinium which resembles in part that of Adams. This scheme is given in Table I.

Table I.

Element.	Radiation.	z.
Actinuranium I	α	92
Uranium V	β	90
Uranium Z	β	91
Actinuranium II	α	92
Uranium Y	β	90
Proto-actinium	α	91
Actinium	β	89
Radio-actinium	α	90

In favour of a scheme of this type are, in addition to the considerations aforesaid, (*a*) the fact that an element of even atomic number is expected from Aston's work to consist of several isotopes, and these in the case of uranium would

* A. Piccard, *Arch. Sci. Phys. nat.* vol. xliv. p. 161 (1917).

† F. Soddy & J. A. Cranston, Proc. Roy. Soc. vol. xciv. A, p. 384 (1918).

‡ E. Q. Adams, J. Amer. Chem. Soc. vol. xlii. p. 2205 (1920).

§ A. Piccard, *Phys. Zeit.* vol. xxiii. p. 1 (1922).

be expected to be radio-active, and (*b*) the fact that the atomic weight of ordinary uranium is in excess of an integer by about 0·18. Only about 0·10 of this can be accounted for by the energy expelled in the transformation of uranium I to radium Ω' (the a of which is assumed to be 206·00). The remaining 0·08 could be explained by the existence of 4 per cent. of an isotope of $a=240$ or 8 per cent. of one of $a=239$. In favour of the accepted view is (*a*) its simplicity, and (*b*) the fact that a dual change, involving the expulsion of α-particles only, is rendered less unlikely than hitherto has been thought by the existence of a dual change involving the expulsion of β-particles only *. Widdowson † and the writer have attempted to distinguish between these views by careful measurements of the relative activities of the products in an old uranium mineral, and have shown how difficult it is to come to a decision from such measurements.

Evidence from two pieces of experimental work, not yet, however, carried out, would distinguish between these opposing theories. The first of these is the determination of the atomic weight of proto-actinium. If this value were found to be a number not expressible by $4n+2$, where n is an integer, the ultimate parent of actinium could not be genetically connected with uranium I or uranium II if α and β be the only particles expelled in a radio-active change. But if it were expressible by $4n+2$, it would be strong evidence in favour of the branch theory, and the value obtained would decide at which body the branching occurred ‡. The second piece of work is a determination of the isotopes of uranium by Aston's methods. In absence of the evidence mentioned above, proof that uranium was a simple element would be evidence in favour of the view that the actinium series arises as a branch ; proof that an isotope of atomic weight say 235 or 239, and in amount of the order of 5 per cent., would be evidence in favour of this isotope being the ultimate parent of the actinium series.

Besides the relation connecting atomic weight and the type of β-change, I have found empirical ones connecting (*a*) atomic weight and the periods of bodies in a $\Sigma\alpha$ change, and (*b*) the periods of the four bodies comprising a β_1

* O. Hahn, *Zeit. Phys. Chem.* vol. ciii. p. 461 (1923); A. S. Russell, 'Nature,' vol. cxi. p. 704 (1923).

† W. P. Widdowson & A. S. Russell, to be published in Phil. Mag. ; A. S. Russell, *loc. cit.*.

‡ A view of the origin of actinium, put forward by S. Meyer (*Zeit. Phys. Chem.* vol. xcv. p. 428 (1920)), reconciles branching at uranium II with an $a=227$ for actinium.

change. The first of these will now be described because it leads to an estimate of the atomic weight of actinium.

Several workers, notably Fajans *, Swinne †, Carruthers ‡, and van den Broek §, have pointed out relations, on the lines of the Geiger-Nuttall relation, connecting atomic weights and the disintegration constants of periods of members of a group of isotopes. It appears to me, however, that simple relations of a quantitative kind can only be obtained if one makes a distinction between the kinds of successive change enumerated in § 2 above. Some of the exceptions to the Geiger-Nuttall relation and to Fajans's α-ray and β-ray rules occur, I think, because this distinction has been lost sight of. My rule is much simpler and much less accurate than most of these, but unless it be fortuitous, which does not appear likely, it is sufficient for the purpose. It is that:

(1) in a $\Sigma\alpha$ change the period of a body is on the average about 800 times that of its successor,

(2) the periods of isotopic members of $\Sigma\alpha$ changes decrease in geometrical progression as the atomic weights decrease in arithmetical progression.

This rule may be illustrated by plotting the logarithms of the periods of the bodies composing the three known $\Sigma\alpha$ changes against their atomic numbers. Three straight lines, approximately parallel, but not equally spaced, one for each series, are obtained. The average slope gives the result (1) above; the distance between the lines the result (2). On the average the periods of uranium, thorium, and actinium members are in the ratio of $(80)^2 : 1 : \frac{1}{63}$. Now if $(80)^2 : 1$ correspond to a difference of atomic weight of 2, $80 : 1$ corresponds to 1, and $63 : 1$ consequently to 0·8. The atomic-weight of radio-actinium is therefore 227·2 or, by Rule VI. (α), 227, and this is also the atomic weight of actinium.

If these considerations be of weight and if α and β be the only particles expelled in radio-active changes, there appears to be no escape from the view that the actinium series originates from an isotope of uranium with an atomic weight either of 235 or 239.

* K. Fajans, *Phys. Zeit.* vol. xiv. p. 950 (1913).
† R. Swinne, *Phys. Zeit.* vol. xiv. p. 145 (1913).
‡ F. G. Carruthers, 'Nature,' vol. xcvi. p. 565 (1916).
§ A. van den Broek, 'Nature,' vol. xciii. p. 480 (1914); and 'Nature,' vol. xcvi. p. 677 (1916).

§ 5. *A Relation between the Periods of Bodies composing a β_1 Change.*

The simple relation connecting bodies in $\Sigma\alpha$ changes may be extended so that the periods of bodies in β_1 changes may be roughly calculated by the following relation :

k, defined by the relation

$$k \cdot \frac{\text{P}_{\text{1st } \alpha\text{-body}}}{\text{P}_{\text{2nd } \alpha\text{-body}}} = \text{P}_{\text{1st } \beta\text{-body}} \times \text{P}_{\text{2nd } \beta\text{-body}},$$

where P represents the period in minutes,

(1) is of the same order (is possibly constant) for any given atomic number, and

(2) diminishes by about $\frac{1}{80}$ for each drop of 2 in atomic number.

The data on which this deduction is based are meagre, nevertheless they do not appear wholly fortuitous.

The values of k, calculated from experimental data, for the β_1 changes UI→Io. Th→ThX, RaA→RaC, and ThA→ThC are given in Table II.

The changes AcA→AcC and RaC″→RaΩ′ are not strictly comparable and so have not been included.

TABLE II.

β_1 Change.	Mean Value of z.	k (min^{-2}).
UI→Io	91	17·7
Th→ThX	89	0·21
RaA→RaC	83	$8{\cdot}7\times10^{-8}$
ThA→ThC	83	$5{\cdot}4\times10^{-7}$

The values of the two last depend upon the periods of RaC″ and ThC″ which are known only from the Geiger-Nuttall relationship*. The real values of k may therefore be a hundred times smaller or larger. It is remarkable, however, that they are of the same order. The value of k for these changes calculated from the relation (2) immediately above is $4{\cdot}3\times10^{-7}$.

§ 6. *The Upper Part of the Actinium Series.*

The scheme which in my view best represents the beginning of the actinium series is given in Table III.

I have followed Piccard† in naming the $z=92$ isotopes

* I have taken them to be 3×10^{-8} sec. and 2×10^{-12} sec. respectively.

† A Piccard, *loc. cit.*

TABLE III.

Position.	Element.	Period.	z.	Radiation.	a.
(1)	Actinuranium I	ca. 8×10^9 years	92	α	239
(2)	Uranium Y_1	28 hours	90	β	235
(3)	Uranium Y_2	ca. 1 minute	91	β	235
(4)	Actinuranium II	$>2\times10^6$ years	92	α	235
(5)	Parent of Proto-actinium	>20 years	90	β	231
(6)	Proto-actinium	$<1{\cdot}2\times10^4$ years	91	α	231
(7)	Actinium	20 years	89	β	227
(8)	Radio-actinium	19·5 days	90	α	227

actinuranium. I differ from both Piccard and Adams * in not placing uranium Y in the position 5, and from Piccard in rejecting uranium V and uranium Z from the scheme. If uranium Y were in the position (5) it contravenes both Fajans's β-ray rule and Rule V. above. Deductions from analogies place it in position (2); experimental evidence does not decide between the two. Piccard's scheme (given in Table I.), in so far as it embraces uranium V and uranium Z, appears untenable, because (1) the existence of uranium V is doubtful †, (2) if it exists, its activity is initially only 2 per cent. of that of uranium X_1 and its period is too great to be the parent of uranium Z ‡, and (3) uranium Z is present only to the extent of 0·35 per cent. of the equilibrium amount of uranium X_1 §. The scheme above, however, postulates a new body, uranium Y_2, with a period of less than a minute (because of Fajans's β-ray rule) corresponding to uranium X_2. The period of actinuranium II is likely to be greater than that of uranium II because of Fajans's α-ray rule. From the relation of § 5 it may be calculated if the period of uranium Y_2 be assumed 1 minute and that of actinuranium I be known. The period of the parent of proto-actinium should be greater than 20 years, because of Rule V.; that of proto-actinium must be as given since the period $1{\cdot}2\times10^4$ years is based on the growth of that element from purified uranium I + uranium II on the assumption that only the short-lived uranium Y lies between. The periods of actinuranium I and actinuranium II are estimated from the considerations of the next paragraph.

* E. Q. Adams, *loc. cit.*

† O. Hahn, *Phys. Zeit.* vol. xxiii. p. 146 (1922); F. Soddy & A. S. Russell, Phil. Mag. vol. xviii. p. 858 (1909).

‡ A. Piccard & E. Stahel, *Phys. Zeit.* vol. xxiv. p. 80 (1923).

§ O. Hahn, *Zeit. Phys. Chem.* vol. ciii. p. 461 (1923); A. S. Russell, 'Nature,' vol. cxi. p. 704 (1923).

§ 7. *The Periods of Actinuranium* I *and Actinuranium* II.

A determination of the ranges of the α-particles of these bodies would enable an estimate of their periods to be obtained from the Geiger-Nuttall relation. This difficult experiment appears to be the only feasible one at present towards that end. It is to be expected, however, that bodies $z=92$, $a=240$; $z=92$, $a=239$, and $z=92$, $a=238$, would have periods in decreasing order of magnitude. The relation which exists between isotopic members of $\Sigma\alpha$ change is inapplicable to these independent isotopes. For want of a better I offer this suggestion. It was shown in fig. 3 above that if the series 220 (the thorium series) had originated from an isotope of uranium, its atomic weight would have have been 240. Suppose then that this body $z=92$, $a=240$, if it existed had the period of thorium, namely $1{\cdot}31\times10^{10}$ years. The body $z=92$, $a=239$, would then probably have a period the geometric mean of this and that of uranium I, *i.e.*, about $7{\cdot}8\times10^{9}$ years. It is perhaps a bold supposition, but the fact that thorium's β-ray product, mesothorium, has a greater period than that expected from the β-ray product of $z=92$, $a=240$, is not an argument against it.

If actinuranium I has a period of $7{\cdot}8\times10^{9}$ years, and if the value of the ratio of the activities of prot-actinium to ionium in equilibrium be accepted as 3 per cent.*, then this body must be present to the extent of $3\times\frac{7{\cdot}8\times10^{9}}{4{\cdot}7\times10^{9}}$, *i.e.*, about 5 per cent. in ordinary uranium.

It may be possible to estimate this percentage accurately in another way by comparing the ratio of uranium to radium in an old mineral obtained directly by analysis with that calculated from the experimental values of the periods of uranium and radium. But at present these are not known sufficiently accurately to give a decision.

The period of actinuranium II may be estimated in three ways:

(1) As $8{\cdot}2\times10^{7}$ years from the relation of § 5 assuming P_{UY_2} to be 1 minute, P_{UY_1} 28 hours, and $P_{AcU\,I}$ $7{\cdot}8\times10^{9}$ years.

(2) As $1{\cdot}4\times10^{7}$ years from the relation

$$\frac{P_{AcU\,II}}{P_{U\,II}}=\sqrt[4]{\frac{P_{U\,I}}{P_{U\,II}}}.$$

(3) As $>2\times10^{6}$ by Fajans's α-ray rule.

* O. Hahn, *Berichte*, vol. lii. B. p. 1812 (1919); W. G. Guy & A. S. Russell, to be published in Journ. Chem. Soc.

§8. *Radio-active Isotopes and Isotopes of Inactive Elements.*

It is of interest to compare some properties of a group of radio-active isotopes with those of a group of isotopes belonging to a common element.

(1) Aston has shown that an inactive element of even atomic number may have isotopes both even and odd in atomic weight. This holds also for radio-active isotopes if actinium have the atomic weight assigned to it in this paper.

(2) The same author has shown that a common element of odd atomic number consists of either one or two isotopes of odd atomic weight. If there be two isotopes they differ in atomic weight by 2. This does not hold at all for radio-active isotopes, *cf.* the isotopes of proto-actinium, bismuth, etc.

(3) Aston's results show that isotopes of even atomic number may differ by 8 and possibly 10 or 12 units of atomic weight. Radio-active isotopes do not appear to differ by more than 8. It is possible that the reason why this number is not exceeded is connected with the stability of the nucleus. None of the isotopes of common element whose existence appears at the moment to contravene this is regarded as firmly established by Aston himself.

I have made use of the difference described under (2) above to put forward a suggestion that is, I think, new. It is obvious that radio-active isotopes may be brought into line with inactive isotopes with respect to this matter if the following be true :—*The end-products of all radio-active series are inactive isotopes.* This idea is tenable, I think, without the implication that such inactive isotopes have been necessarily produced as a result of radio-active disintegration.

Simple analysis has shown that elements of atomic number 80, 79, and lower numbers do not appear to have isotopes in the radio-active series. Possible end-products are thus confined to isotopes of atomic numbers 83, 82, and 81. Some of these have been shown marked with a black circle in fig. 2 above. In series 221 the lower portion of the $\beta_1\,\beta_2$ path is incomplete. At the $z=83$, $a=209$, body a branch may occur, but since the product $z=84$, $a=209$, is not possible because of Rule IV., this branch of the series may end at $z=83$, $a=209$; the main product of the branching $z=81$, $a=205$, cannot be transformed (as symmetry indicates) into $z=82$, $a=205$, because of Rule IV., and consequently the main portion of the series ends at $z=81$, $a=205$. It is possible also for part of series 219 to end at $z=81$, $a=203$, without contravening any of the guiding rules of this paper.

Possible end-products are tabulated in Table IV. In the third column are given the atomic weights of possible end-products, and in the fourth those *to be expected* experimentally from Aston's investigations.

TABLE IV.

Element.	z.	Values of a deduced from the series				Values of a to be expected experimentally.
		222	221	220	219	
Thallium ...	81	—	205	—	203	203, 205
Lead	82	206, 210	209?	208, 208	207	206, 208, 210, etc.
Bismuth ...	83	–	209	—	—	209

It is assumed here that thallium's inactive isotopes are 203 and 205, because this element has an odd atomic number and its experimental atomic weight is 204. Bismuth has probably only one inactive isotope, 209; if it has a second it is more likely to be 207 than 211. The postulates of Table IV. are not inconsistent with the research of Soddy and Hyman *, who found a trace of thallium but no bismuth in a specimen of thorite containing only 0·85 per cent. U_3O_8, or with that of Merton †, who found a trace of thallium with a spectrum slightly different from that of ordinary thallium in pitchblende residues. The known limits of atomic weights of radio-active isotopes are given for reference in Table V.

TABLE V.

Element.	z.	Limits of a.	a of longest-lived or common isotopes.
Uranium	92	239–231	234, 238, 239?, 235?
Proto-actinium	91	235–231	231
Thorium	90	235–227	232, 233?
Actinium.....	89	227–223	227
Radium	88	226–223	226
Radon	86	222–219	222
Polonium	84	218–210	210
Bismuth	83	214–207?	209, 207?
Lead	82	214–206	206, 208, 210, 209?, 207?
Thallium.....	81	210–203	203, 205
Mercury	80	204–197	197–200, 202, 204

* F. Soddy & H. Hyman, Journ. Chem. Soc. vol. cv. p. 1402 (1914).
† T. Merton, Proc. Roy. Soc. vol. xcvi. A. p. 394 (1929).

The rarest of all known longest-lived or common isotopes except radon is polonium. Analysis shows that no isotope of longer period is likely. The shortness of the period suggests that polonium can never be obtained in sufficient concentration to be treated as a common element. Its existence, detectable only by the methods of radio-activity, helps to justify the assumption often made that missing elements have not been discovered, less because the search for them has been poor than for the fact that, if formed at all in nature, they are too unstable to be detected by the balance.

Experimental work on uranium Y_2, on the hypothetical series 221, and on the nature of branching in all series, is in progress.

Summary.

(1) In discussing analogies or general relations between the three disintegration series it has been found most advantageous to regard a series as made up of three types of successive changes, and to keep these distinguished.

(2) It is deduced that there are probably only four independent disintegration series. The path of disintegration of each of these is given.

(3) Simple empirical relations between the periods of bodies comprising a $\Sigma\alpha$ change and between those comprising a β_1 change are given.

(4) It is concluded that actinium arises from an isotope of uranium with an atomic weight of 239, present to the extent of about 5 per cent. Estimates of the periods of the bodies which precede proto-actinium in its series are given.

(5) Radio-active isotopes are compared with those of inactive elements and a suggestion with regard to the existence of end-products other than those of isotopes of lead is put forward.

Dr. Lee's Laboratory,
Christ Church, Oxford.
June 1923.

LXXIII. *Motion of Electrons in Helium.* *By* J. S. Townsend, *M.A., F.R.S., Wykeham Professor of Physics, Oxford, and* V. A. Bailey, *M.A., D.Phil., The Queen's College, Oxford* *.

1. In previous communications to the Philosophical Magazine † we have described a method of determining the motion of electrons in various gases, and we have recently used the method to determine the motion in helium.

The experimental part of the investigation consists in determining the velocity W of a stream of electrons in the direction of a uniform electric force Z, and the velocity of agitation u. For this purpose we found it convenient to use two instruments, one being suitable for the measurement of the velocities in gases where the stream of electrons has a moderate lateral diffusion, and the other suitable for gases like argon where the lateral diffusion of the stream is very large. A full description of these two instruments is given in the account of our researches on Argon and Hydrogen ‡. The former instrument was arranged so that a stream of electrons, after passing through a narrow slit, was received on three electrodes at a distance of 4 centimetres from the slit, and in the latter the receiving electrodes were 2 centimetres from the slit.

With helium, for the smaller values of the ratio (Z/p) of the electric force Z to the pressure of the gas p, the determinations were made with the instrument in which the electrodes were 4 centimetres from the slit, and for the larger values of the ratio Z/p the determinations were made with the other instrument. For certain ranges of forces and pressures it was possible to make determinations with both instruments, and concordant results were obtained.

2. In these experiments two specimens of helium were used—one which was obtained from a natural source, and the other obtained by heating thorianite. The velocities of the electrons were found to be almost exactly the same in the two specimens.

In each case the gas was purified by means of charcoal cooled by liquid air. The apparatus was tested by exhausting it to 1/100 of a millimetre pressure, and no leak was detected by a McLeod gauge when the pressure

* Communicated by the Authors.

† Phil. Mag. December 1921, March, June, and November 1922.

‡ Phil. Mag. November 1922.

was observed at intervals during several weeks. The connexion to the pump and mercury gauges was made through tubing cooled by liquid air, in order to reduce any effect which might be due to mercury vapour. It was not found, however, that this precaution to remove mercury vapour made any appreciable difference to the results of the experiments, which may have been due to the fact that all the electrodes and metal rings in the apparatus were silver-plated and absorbed mercury vapour.

3. The agreement between the values of W and k, obtained with different forces and pressures when the ratio Z/p was constant, shows that there could have been no notable effect due to an impurity unless the amount of the impurity were proportional to the pressure of the gas. It therefore follows from the results of the experiments that the mercury vapour had no appreciable effect, as it would be impossible for the vapour pressure in the apparatus to be proportional to the gas pressure. This result was tested with considerable care after most of the observations were made in the ordinary manner. The method of removing mercury vapour recommended by Professor Merton was used, which consists in absorbing the vapour by silvered glass tubing, the silver being deposited by a discharge from silver electrodes. Before making these experiments the whole apparatus was taken down and all the electrodes and metal rings in the measuring instruments were replated with silver so as to expose the gas to a large surface of pure silver. The instrument was then connected to the pump and gauges through the silvered glass tubing and a tap which was turned off immediately after the pressure in the apparatus had been measured. The apparatus was exhausted and washed out with helium to remove traces of impurities. The helium used in these experiments was pumped into a tube filled with charcoal which was cooled with liquid air, and the gas was kept in the charcoal tube for an hour before it was admitted to the apparatus. Measurements of W and k were made with the gas at the pressures 20, 10, and 5 millimetres, and they were found to be almost exactly the same as those made in the ordinary manner when no special precautions were taken to remove mercury vapour.

4. The results of the experiments are given in Table I., where p is the pressure of the gas in millimetres, Z the electric force in volts per centimetre, W the velocity of the electrons in the direction of the electric force in centimetres per second, and k the factor by which the energy of

agitation of the electrons exceeds the energy of agitation of an equal number of molecules of a gas at 15° C.

TABLE I.

p.	Z.	Z/p.	k.	$W \times 10^{-5}$.
240	3·15	·0131	1·75	1·07
160	2·1	·0131	1·79	1·14
240	6·3	·0262	...	1·50
160	4·22	·0263	2·52	1·56
120	4·2	·035	3·01	1·73
160	8·43	·0527	3·93	2·15
120	6·3	·0525	3·90	2·12
80	4·22	·0527	3·76	2·30
120	12·6	·105	6·65	2·92
80	8·43	·105	6·46	3·06
80	16·9	·21	12·0	4·08
40	8·4	·21	11·6	4·00
20	4·22	·21	11·8	3·95
20	8·45	·42	22·8	5·3
10	4·2	·42	23·3	5·0
20	16·9	·845	44·0	7·6
10	8·4	·84	45·3	7·4
10	16·9	1·69	91	11·4
7·5	12·6	1·68	93	11·5
7·5	21·0	2·8	134	16·4
5	16·9	3·38	145	19·6
5	21·0	4·2	158	25·2
5	26·5	5·3	177	32·0

The numbers given in the table are the means of several experiments which were in good agreement. Those corresponding to the lower values of the ratio Z/p from ·0131 to ·105 were obtained with the instrument in which the receiving electrodes were 4 centimetres from the slit, and most of the other numbers were obtained with the other instrument.

The curves given in figs. 1 and 2 represent the values of W and k for helium in terms of the ratio Z/p.

Fig. 1.

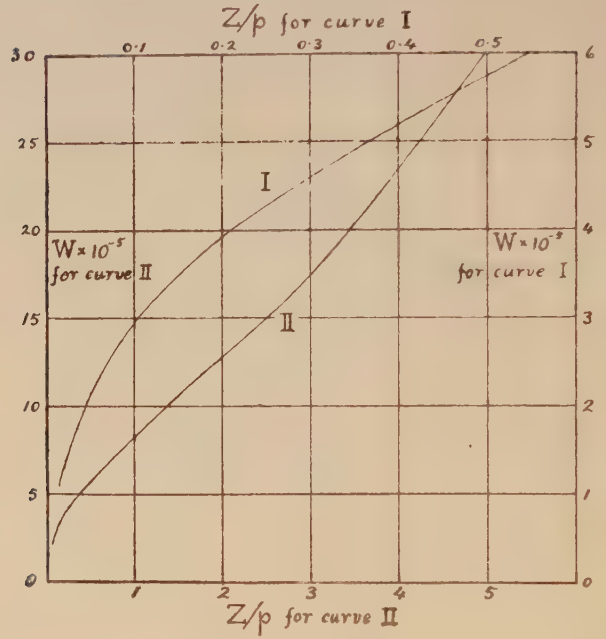
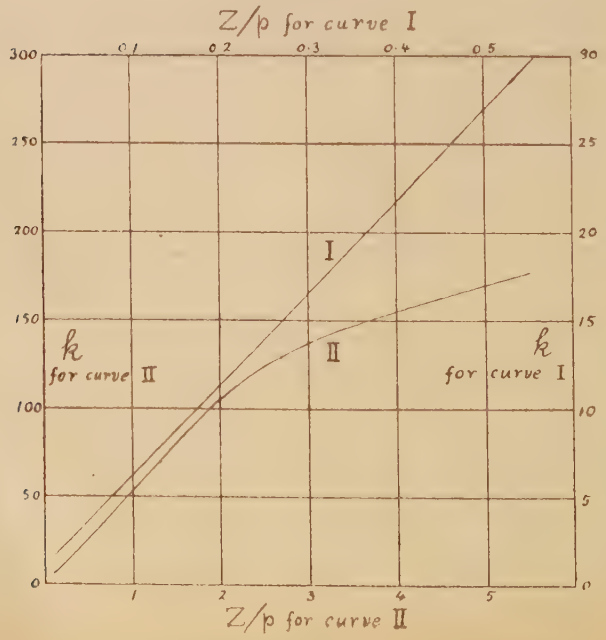

Z/p for curve I
0·1
0·2
0·3
0·4
0·5
30
25
20
15
10
5
0
6
5
4
3
2
1
0
I
II
W × 10⁻⁵ for curve II
W × 10⁻⁵ for curve I
1
2
3
4
5
Z/p for curve II

Fig. 2.


Z/p for curve I
0·1
0·2
0·3
0·4
0·5
300
250
200
150
100
50
0
90
25
20
15
10
5
0
I
II
k for curve II
k for curve I
1
2
3
4
5
Z/p for curve II

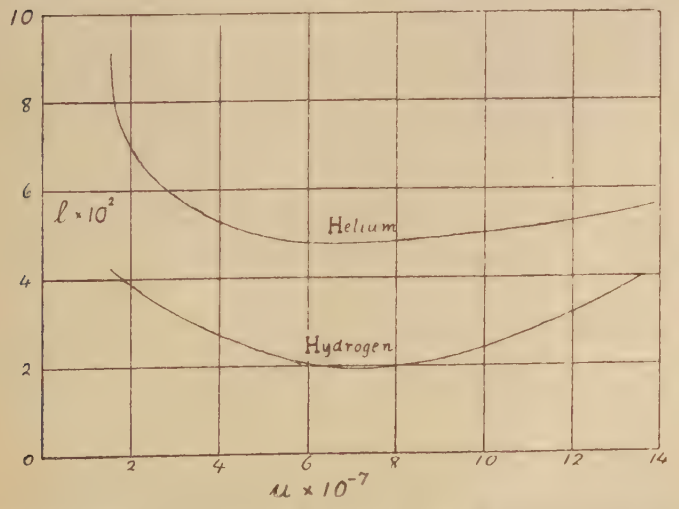
5. The velocity of agitation u, the mean free path l, and the proportion λ of the energy of an electron which is lost in a collision, found from the above values of k and W by the formulæ $u = 1{\cdot}15 \times 10^7 \sqrt{k}$, $W = {\cdot}815 Zel/mu$, and $\lambda = 2{\cdot}46 W^2/u^2$, are given in Table II. The values of l are for the gas at 1 millimetre pressure.

TABLE II.

Z/p.	k.	$W \times 10^{-5}$.	$u \times 10^{-7}$.	$l \times 10^2$.	$\lambda \times 10^4$.
·013	1·77	1·11	1·53	9·14	1·30
·02	2·12	1·33	1·68	7·8	1·56
·05	3·68	2·14	2·21	6·6	2·3
·1	6·2	2·96	2·87	5·95	2·6
·2	11·3	3·93	3·87	5·3	2·55
·5	27	5·74	5·96	4·8	2·3
1·0	53	8·25	8·4	4·85	2·4
1·5	79·5	10·5	10·3	5·0	2·6
2·0	105	12·7	11·8	5·25	2·85
2·5	124	15·0	12·8	5·4	3·4
3·0	137	17·5	13·5	5·5	4·15
4·0	152	23·5	14·2	5·85	6·75
5·0	172	30·2	15·1	6·4	9·8

6. In order to compare the mean free paths of electrons in helium with the mean free paths in hydrogen, the values of l for the two gases at a millimetre pressure corresponding

Fig. 3.



to velocities of agitation u from $1{\cdot}5 \times 10^7$ to 14×10^7 cms. per sec. are given by the curves in fig. 3. The free path of

an electron in helium is about twice as long as the free path in hydrogen, but if the free paths were deduced from the viscosities of the gases the lengths would be in the proportion of about 3 to 2. It will be noticed that there is an increase in the mean free path in helium from 5×10^{-2} to 9×10^{-2} cm. when the velocity of the electron decreases from 5×10^{7} to $1{\cdot}5 \times 10^{7}$ cms. per sec. Judging from the direction of the curve, it is probable that there would be a further increase in l if the velocity were reduced below the value $1{\cdot}5 \times 10^{7}$. This velocity corresponds to a potential difference of ·065 volt.

In helium the mean free path is nearly constant for velocities between 10^{8} and 5×10^{7} cms. per sec., but in argon the mean free path increases from about 18×10^{-2} cm. when the velocity is 10^{8} to about 110×10^{-2} cm. when the velocity is 5×10^{7}. Thus the increase in the free path as the velocity diminishes occurs for smaller velocities in helium than in argon.

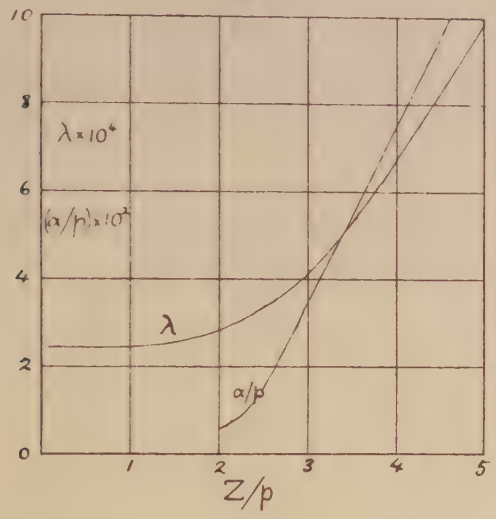
Experiments are at present being made with neon, but a sufficient number of results have not yet been obtained to compare this gas with helium and argon.

7. The numbers in the last column of Table II. show that the proportion of the energy of an electron lost in a collision is nearly constant for the range of velocities from 2×10^{7} to 10^{8} cms. per sec. This proportion is about $2{\cdot}5 \times 10^{-4}$, and is approximately the same as the average proportion of energy which would be lost when a small particle of mass m collides with a perfectly elastic sphere of mass M if m/M were equal to the ratio of the mass of an electron to the mass of a helium atom. This, however, does not justify the hypothesis that the helium atom resembles an elastic sphere, since the variations in l are not consistent with such an hypothesis.

For velocities greater than 10^{8} and values of Z/p greater than 2, the values of λ increase considerably. It may be seen that this increase is due to ionization by collision by comparing the curves representing λ and α/p in terms of Z/p. These curves are given in fig. 4. The quantity α has the usual signification, namely, the average number of molecules ionized by an electron in moving through a centimetre in the direction of the electric force. The values of α/p which are given are those obtained in an investigation made with Mr. Ayres of the effects of ionization by collision, which will be published shortly. The same specimens of helium were used in all these determinations,

and the same precautions were used to eliminate mercury vapour.

Fig. 4.



Note on the Determination of W.

In the accounts of our previous experiments we have mentioned that the velocities W determined with different forces and pressures, when the ratio Z/p is constant, are practically the same, provided the streams of electrons are of moderate divergence. But with very widely diverging streams the values obtained for W are smaller, when calculated by the same method. We regarded this as being probably due to part of the stream being deflected by the transverse magnetic force beyond the boundary of the receiving electrodes. Further experiments have shown that the discrepancy would not be eliminated by increasing the area of the receiving electrodes, and that it arises from a secondary effect on the stream due to the electrodes, which becomes appreciable when the coefficient of diffusion K of the electrons is very large compared with the velocity W in the direction of the electric force. The discrepancy is seen by considering the results of the set of experiments with hydrogen given in Table III., which were made with an instrument where the receiving electrodes were 4 centimetres from the slit. In this table R is the ratio of the charge received by the central electrode E_2

to the total charge received by the three electrodes E_1, E_2, and E_3 when no magnetic force is acting.

TABLE III.

p.	Z.	Z/p.	R.	$W \times 10^{-8}$.	k.
10	34	3·4	·53	21·5	20·4
5	17	3·4	·394	21·8	20·2
2·53	8·5	3·36	·29	21·5	20·2
1·24	4·25	3·42	·214	19·0	20·1

The ratio R is a function of W/K or $41\,Z/k$, and the relation connecting R and Z/k is given by means of a curve as shown in fig. 3 on p. 1039 of the Philosophical Magazine of November 1922. The experiments show that when the ratio Z/p is constant, the values of W obtained with different forces and pressures are in good agreement when R is greater than ·29, but with a very widely diverging stream (R = ·21) the value of W is about 12 per cent. less than that obtained when R is greater than ·29.

In this method of measuring the velocity W from observations of the magnetic deflexion of the centre of the stream, the determinations should be independent of the divergence of the stream when the divergence is not very large, and several sets of experiments have shown that when Z/p is constant the velocities obtained with different forces and pressure are practically the same when R is greater than ·24 or Z/k greater than ·28. These values are therefore the right ones, and the correction which must be applied when the stream is very divergent is easily found experimentally. This conclusion is in agreement with the fact that the four determinations of k given in Table III. are the same, since this indicates that the velocity in the last experiment should be the same as in the others.

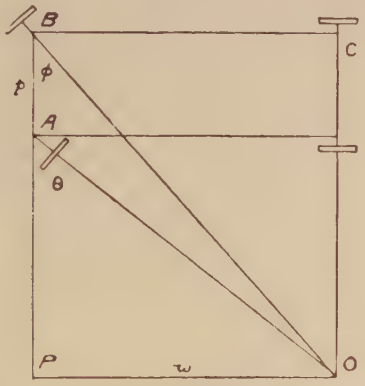
With the second instrument, where the receiving electrodes are 2 centimetres from the slit, it will be seen from the curves giving R in terms of Z/k that R is ·33 when Z/k is ·28. Thus, although this instrument is convenient for measuring k when the streams of electrons are very widely divergent, it is necessary to correct the values of W when R is less than ·33. In the experiments with monatomic gases the ratios R were in many cases less than this, and the corresponding values of W which we have given for argon and helium are the corrected values.

LXXIV. *The Automobile Steering Linkage.* *By* P. Cormack, *F.R.C.Sc.I., Engineering Dept., College of Science, Dublin* *.

1. *The Concurrent Equation.*

IN order that the wheels of a motor-car may roll without side-slip of the tyres on the ground, the plans of the axes of the road wheels must be concurrent. In fig. 1 the axes AO, BO, of the steering wheels, and the common axis CO of the rear wheels, intersect at O: the car turns about O as centre, this being the unique turning-point which allows each wheel to pursue its natural rolling path. There is now a simple relation between θ and ϕ, the angles of deflexion of

Fig. 1.



the inner and the outer steering wheels. Let p denote the pitch of the steering pivots AB, and let w denote the wheel-base PO. From fig. 1,

$$AB = PB - PA = PO \cot\phi - PO \cot\theta,$$

whence
$$\cot\phi - \cot\theta = p/w. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It follows readily by the method of indirect proof that when the deflexions of the steering wheels satisfy (1) the wheel axes are concurrent.

For very small values of θ and ϕ, equation (1) becomes

$$1/\phi - 1/\theta = p/w,$$

or, in other words, we have,

$$\underset{\theta=0}{\text{Lt.}}\,[(\theta-\phi)/\theta\phi] = p/w. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

* Communicated by the Author.

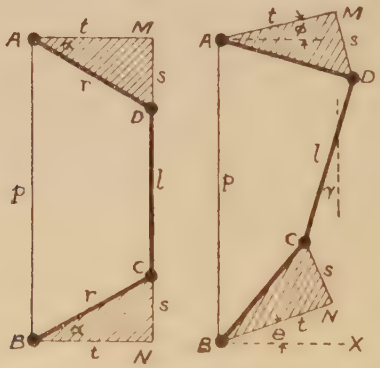
2. *Property of Quadrilateral.*

The usual automobile steering linkage is shown in figs. 2 and 3. In its mean position, fig. 2, A and B are the fixed steering pivots about which the stub-axles of the steering wheels with their attached steering arms AD, BC, turn. CD is the coupler or tie-rod joining the ends of the steering arms.

Let us drop AM, BN, perpendicular to CD, cutting CD produced in M and N, and consider now the triangles ADM, BCN as plates attached to and partaking of the movements of the steering arms. We shall use t to denote AM the depth of the steering quadrilateral, and s to denote the length MD.

Fig. 3 shows the steering linkage with the arm AD (and in consequence AM and the outer steering wheel) deflected

Fig. 2. Fig. 3.



through an angle ϕ, and with the arm BC (and in consequence BN and the inner steering wheel) deflected through an angle θ. Denoting by γ the angle which CD makes with AB, we have by projection on AB

$$t\sin\theta + s\cos\theta + l\cos\gamma + s\cos\phi - t\sin\phi - p = 0,$$

while by projection on BX we have

$$t\cos\theta - s\sin\theta + l\sin\gamma - s\sin\phi - t\cos\phi = 0.$$

Writing out the value of $\cos^2\gamma$ from the first equation and the value of $\sin^2\gamma$ from the second equation and adding the two results, we eliminate γ and obtain the following equation :

$$s^2[1-\cos(\theta-\phi)] - s[p(2-\cos\theta-\cos\phi) + 2t\sin(\theta-\phi)] + pt(\sin\theta-\sin\phi) - t^2[1-\cos(\theta-\phi)] = 0. \quad . \quad (3)$$

If both s and t be taken negative, this equation is also true for the external quadrilateral in which the length of the the coupler is greater than the pitch of the steering pivots and in which the coupler is placed in front of the cross-member which carries the steering pivots.

It may be noted that the ratio s/t is $\tan\alpha$, where α is the angle made by the plan of the steering arm with the path of the wheel.

It is evident that for given values of p, w, and t (or s) we can design the steering quadrilateral to give correct steering for any assigned value of θ; for by determining ϕ from (1) and then substituting in (3) we can find s (or t) and hence draw the gear.

3. *Property of Quadrilateral, neglecting Effect of Coupler Obliquity.*

Neglecting the effect of coupler obliquity is equivalent to taking s and t to be negligible in comparison with p. Dividing (3) throughout by p and neglecting the terms containing s/p and t/p, we obtain s/t or

$$\tan\alpha=(\sin\theta-\sin\phi)/(2-\cos\theta-\cos\phi),$$

an equation which may be readily deduced directly.

When θ and ϕ satisfy (1) the limiting value for $\theta=0$ of $(\sin\theta-\sin\phi)/(2-\cos\theta-\cos\phi)$ is p/w. For writing the fraction equal to

$$\frac{(\theta-\frac{1}{6}\theta^3+\ldots.)-(\phi-\frac{1}{6}\phi^3+\ldots.)}{2-(1-\frac{1}{2}\theta^2+\ldots.)-(1-\frac{1}{2}\phi^2+\ldots.)}$$

we get its limiting value to be the same as that of

$$(\theta-\phi)/(\tfrac{1}{2}\theta^2+\tfrac{1}{2}\phi^2)$$

or

$$\frac{2(\theta-\phi)}{\theta\phi}\Big/\left[\frac{\theta-\phi}{\theta\phi}\cdot(\theta-\phi)+2\right],$$

which, making use of (2), is p/w.

Hence for correct steering to obtain with small deflexions of the steering wheels we must make $\alpha=\tan^{-1}p/w$ *or, what is equivalent, we must make the steering arms converge to a point midway between the front and back axles.*

4. *Limiting Value of Steering-arm Angle, Obliquity of Coupler included.*

The accurate gear equation is (3). Dividing (3) throughout by $\sin\theta-\sin\phi$ and taking the limiting form of each

term as θ approaches zero and when θ and ϕ satisfy (1), we get

$$0 \cdot s^2 - s(p \cdot w/p + 2t) + pt - 0 \cdot t^2 = 0,$$

from which

$$s/t = p/(w+2t), \quad \ldots \ldots \quad (4)$$

or again

$$s/t = (p-2s)/w = l/w.$$

Hence for correct steering to obtain when running straight or in a curve of very large radius, we must make $\alpha = \tan^{-1} l/w$ *or* $\tan^{-1} \frac{1}{2} p/(\frac{1}{2}w+t)$, *that is, the steering arms must converge to a point at a distance t from the centre of the car—in front of the centre for an external quadrilateral for which t is negative, and to the rear of the centre for an internal quadrilateral for which t is positive.*

5. *An Approximate Equation for Design of Quadrilateral.*

An empirical equation simpler than (3) and sufficiently accurate for practical purposes, suggested by the above form for (3) when $\theta = 0$, is

$$\tan \alpha = \frac{p}{2(w+2t)} [1 + \cos 1{\cdot}63\theta]. \quad \ldots \quad (5)$$

It must be remembered that a negative value for t must be introduced in (5) for an external quadrilateral. Instead of the depth t of the quadrilateral, the length r of the steering arm may be used without appreciable error; so that we have for a working formula

$$\tan \alpha = \frac{p}{2(w+2r)} [1 + \cos 1{\cdot}63\theta]. \quad \ldots \quad (6)$$

As an illustrative example we shall design a gear for a car having a wheel-base of 120 inches; pitch of steering pivots 48 inches; length of steering arm 7 inches; wheel axes to be concurrent at 30° deflexion of inner wheel.

We have, substituting in the design formula (6)

$$\tan \alpha = \frac{48}{268} (1 + \cos 48^\circ 54') = {\cdot}2969,$$

or $\alpha = 16\frac{1}{2}^\circ$. (If an external quadrilateral were required we should have

$$\tan \alpha = \frac{48}{2(120-14)} (1 + \cos 48^\circ 54') = {\cdot}3752,$$

which gives $\alpha = 20\frac{1}{2}^\circ$.)

For small deflexions, this gear will give the steering-wheel

axes intersecting at a distance w_0 behind the front axle, given by

$$\tan\alpha = \frac{p}{2(w_0+2r)}(1+\cos 0°),$$

from which $w_0 = 147$ inches.

Taking the maximum angle of lock for this car to be 40°, for this deflexion of the inner steering wheel the front stub axles intersect at a distance w_m behind the front axle given by

$$\tan\alpha = \frac{p}{2(w_m+2r)}(1+\cos 1{\cdot}63\times 40°),$$

from which $w_m = 100$ inches.

The gear thus gives the steering-wheel axes intersecting 27 inches behind the back axle at the beginning of the steering movement, intersecting on the back axle when the deflexion of the inner wheel is 30°, and intersecting 20 inches in front of the rear axle at the end of the steering movement. To design a gear to give the point of intersection of the steering-wheel axes as much behind the back axle at the beginning of the steering movement (deflexion 0°) as it is in front of the rear axle at the end of the steering movement (deflexion ψ), we have from (6)

$$w_0+2r = p(1+\cos 0°)/2\tan\alpha,$$

$$w_m+2r = p(1+\cos 1{\cdot}63\psi)/2\tan\alpha,$$

and since we require $w = \frac{1}{2}(w_0+w_m)$ we get

$$w+2r = p(3+\cos 1{\cdot}63\psi)/4\tan\alpha$$

or

$$\tan\alpha = \frac{p}{4(w+2r)}[3+\cos 1{\cdot}63\psi]. \quad . \quad . \quad . \quad (7)$$

With the above dimensions this gives a setting of 17° for the steering arm angle α.

The design formula (5) may be used to calculate the performance of an existing gear. Take, for example, a car having $p = 52$ ins., $w = 100$ ins., $t = 6$ ins., $s = 2$ ins. To find the angle θ at which correct steering obtains, we have $\tan\alpha = s/t = 2/6$, so that substituting in (5)

$$1/3 = 52(1+\cos 1{\cdot}63\theta)/2(100+12),$$

whence $\theta = 39°$.

Historically, the gear with steering arms converging to the centre of the back axle is of interest. For a general notion of the performance of this gear, for which $\tan\alpha = p/2w$,

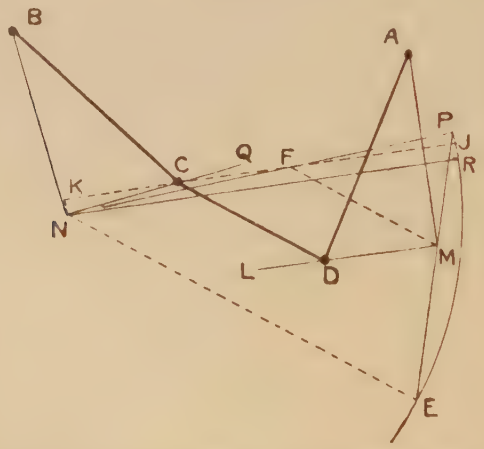
we may neglect t in (5). We then obtain $1 = 1 + \cos 1{\cdot}63\theta$, from which $\theta = 55°$, an angle much outside the working range usual in motor-car practice.

The inclinations sometimes given to steering pivots and to stub-axles do not appreciably affect the design of the linkage; the actual angle of deflexion of the stub-axle is sensibly the same as the angle in plan, while in plan the path of the steering-arm end is an ellipse which differs but little from the actual circular path.

6. *Graphical Solution of Design Problem.*

It may not be without interest to give a graphical solution of (3) when the known magnitudes are θ, ϕ, and t. The given data enable us to draw the lines AB, AM, BN, and the perpendiculars MDL, NCQ at M and N figs. 3 and 4. The

Fig. 4.



problem evidently now is to fit in CD between ML and NQ so that NC = MD and NC + CD + DM = AB. From N draw NR parallel to ML and make NR equal to AB, the pitch of the steering pivots. Draw NP bisecting the angle QNR and from R drop RP perpendicular to NP. Join PM.

With centre N and radius NR describe a circle cutting PM in E. Join EN and through M draw MF parallel to EN cutting NP in F. Through F draw FC parallel to MD cutting NQ in C. Through C draw CD parallel to MF cutting ML in D. Join BC, AD. Then BCDA is the required linkage.

The construction evidently makes NC = CF = MD. Draw

NK parallel to RP and let CF be produced to meet RP in J and NK in K. The triangles PNE, PFM are similar, therefore

$$NE : FM :: NP : FP.$$

Again, since the triangles PNR, PFJ are similar,

$$NP : FP :: NR : FJ.$$

Hence

$$NE : FM :: NR : FJ.$$

And since NE=NR we therefore have FM=FJ. But FM=CD, hence CD=FJ.

Since NC=CF and the angle FNK is a right angle, the circle centre C which passes through F and N will pass through K so that KC=NC.

Hence finally we have

$$NC + CD + DM = KC + CF + FJ = KJ = NR = AB$$

as required.

LXXV. *On the Decay of Vortices in a Viscous Fluid.* By G. I. Taylor, *F.R.S.**

A NUMBER of problems have been solved in which the rate of decay of small oscillations or waves in a viscous fluid has been found, but the simplification brought about by considering only small motions excludes many of the most important problems of fluid motion. On the other hand, when the complete equations of motion involving terms containing the square of the velocity have been used very few solutions have been obtained. Certain problems in steady motion †, problems concerning the two-dimensional motion of a viscous liquid when it is symmetrical about an axis ‡, and problems concerning laminar motion parallel to a plane § probably complete the list.

The object of the present paper is to draw attention to a class of cases in which solutions of the complete equations

* Communicated by the Author.

† Flow through a pipe or between concentric cylinders.

‡ See Taylor, 'Reports and Memoranda of the Advisory Committee for Aeronautics, 1918 19.' Also Terazawa, 'Report of Aeronautical Research Institute,' Tokio, 1922.

§ See Lamb, 'Hydrodynamics,' chap. xi.

of motion, including the "inertia" terms, may be obtained for two-dimensional viscous flow.

If ψ represents the Stream Function so that the components of velocity are

$$u = -\frac{\partial \psi}{\partial y}, \qquad v = \frac{\partial \psi}{\partial x},$$

the vorticity at any point is

$$\zeta = \nabla^2 \psi.$$

The equation of motion may be written

$$\left(\frac{\partial}{\partial t} - \frac{\partial \psi}{\partial y}\frac{\partial}{\partial x} + \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y} - \nu \nabla^2\right)\nabla^2 \psi = 0, \quad . \quad (1)$$

where ν is the coefficient of kinematic viscosity. Now consider functions which satisfy the equation

$$\nabla^2 \psi = \mathrm{K}\psi,$$

where K is a constant. These functions will also satisfy (1) if

$$\frac{\partial \psi}{\partial t} - \nu \mathrm{K}\psi = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the "inertia" terms

$$-\frac{\partial \psi}{\partial y}\frac{\partial (\nabla^2 \psi)}{\partial x} + \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}(\nabla^2 \psi)$$

vanishing because the stream lines are also lines of constant vorticity. Equation (2) is satisfied if

$$\psi = \psi_1 e^{\nu \mathrm{K} t},$$

where ψ_1 is a function of x and y only. Hence, if ψ_1 is a solution of

$$\nabla^2 \psi_1 = \mathrm{K}\psi_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\psi = \psi_1 e^{\nu \mathrm{K} t}$$

is a solution of (1).

Analogy with the theory of vibrating membranes.

The equation of motion for a vibrating membrane is

$$\nabla^2 z = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where z is the displacement at any point from the position

of equilibrium and c depends only on the tension and mass of the membrane. If the membrane vibrates in simple harmonic motion of period T, (4) becomes

$$\nabla^2 z + \frac{4\pi^2 z}{c^2 T^2} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This equation is of the same form as (3), and the amplitude of vibration of the membrane may be taken to represent ψ if

$$K = -4\pi^2/c^2T^2.$$

It appears therefore that if a solution of the problem of a vibrating membrane has been obtained so that the period and contours, or curves of equal displacement of the membrane, have been determined, a problem of viscous motion has also been solved in which the stream lines are the same as the contours of the vibrating membrane. The velocity of the flow dies down exponentially so that it is reduced in the ratio $e:1$ in time $\frac{c^2T^2}{4\pi^2}$.

Though the analogy is exact, so far as it goes, it should be borne in mind that in general the boundary conditions in the two cases are different. In the case of a membrane held round its edges, for instance, the boundary condition is $z=0$, while for the viscous fluid both ψ and $\partial\psi/\partial n$ must be zero at a solid wall. Another point of difference is that if two different solutions have been obtained in the membrane problem, they can be superposed. This is not true in the case of the viscous motion problem because the "inertia" terms in (1), namely,

$$-\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}(\nabla^2\psi) + \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}(\nabla^2\psi),$$

vanish when ψ is a solution of

$$\nabla^2\psi = K_1\psi, \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

or when it is a solution of

$$\nabla^2\psi = K_2\psi, \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

but they do not vanish when ψ is a sum of solutions of (6) and (7).

Decay of a system of eddies rotating alternately in opposite directions and arranged in a rectangular array.

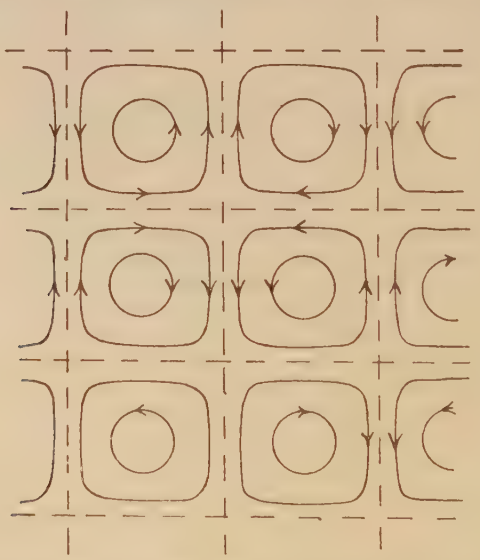
As an example of the use of this type of solution of the equations of motion of a viscous fluid the function

$$\psi_1 = \mathrm{A} \cos \frac{\pi x}{d} \cos \frac{\pi y}{d},$$

which is a solution of (3), may be considered. In this case K is $2\pi^2/d^2$, so that the corresponding solution of (1) is

$$\psi = \mathrm{A} \cos \frac{\pi x}{d} \cos \frac{\pi y}{d} e^{-\frac{2\pi^2 \nu t}{d^2}}.$$

This represents a system of eddies arranged in a square pattern, each rotating in the opposite direction to that of its four neighbours. d is the length of the side of a square containing one complete eddy, and such a system is reduced in intensity in the ratio $e:1$ in time $d^2/2\pi^2\nu$. The intensity



Stream lines for system of eddies dying down under the action of viscosity.

of such a system of eddies, each 1 cm. in diameter, for instance, would be reduced to 1/eth in $\frac{1}{3}$ of a second owing to the action of viscosity if the fluid were air, or $4\frac{1}{2}$ seconds in the case of water. The stream lines for this system of eddies are shown in the accompanying figure.

LXXVI. *The Period of Protactinium.*

By JOHN H. MENNIE, *B.A.* (*Vanc.*), *B.Sc.* (*Oxford*) *.

PROTACTINIUM, the parent of actinium, was discovered by Soddy and Cranston †, and independently by Hahn and Meitner ‡. Shortly after the present investigation was commenced, Hahn and Meitner published an account of a similar research §. Their result has been confirmed by the writer, using a somewhat different method.

Hahn and Meitner separated the protactinium which had formed in a known interval of time in a uranium preparation assumed to be originally free from it, and compared its α-activity with that of the uranium. They used three separate preparations: 141 grams uranyl nitrate, 20 years old; 86 grams uranyl nitrate, 52 years old; and 33·1 grams sodium uranate plus 21 grams uranyl acetate, 60 years old. The chemical history of these can hardly have been known with absolute accuracy but, as a test experiment with a sample of ordinary commercial uranyl nitrate, freshly prepared, showed practically no protactinium present, they concluded that the ordinary commercial methods of purification are sufficient to remove it all, and hence that their specimens were originally free from protactinium. The uranyl nitrate solution was precipitated with ammonia and the uranium got into solution again by boiling with ammonium carbonate. The precipitate was then dissolved in hydrofluoric acid and a few milligrams of tantalic acid added. The solution was filtered, the filtrate evaporated to dryness, and boiled with aqua regia to remove iron, zirconium, etc., and any traces of uranium and polonium. The tantalum, which was insoluble in the aqua regia, contained the protactinium. This was filtered out and spread on an aluminium plate and its α-activity compared with that of a known weight of uranium oxide spread on a similar plate. The three results obtained were 12,800, 11,300, and 12,300 years; giving a mean value of 12,000 years for the half-value period.

In the present investigation the period of the protactinium was determined by comparison with that of ionium. The protactinium and also the ionium which had grown in the same time in a uranium preparation originally freed from these elements, were separated and their activities measured.

* Communicated by Prof. F. Soddy, M.A., F.R.S.

† Proc. Roy. Soc. A, vol. xciv. p. 384 (1918).

‡ *Phys. Zeit.* vol. xiv. p. 208 (1918); vol. xx. p. 127 (1919). *Berichte*, vol. lii. p. 1812 (1919).

§ *Berichte*, vol. liv. p. 69 (1921).

Assuming Hahn and Meitner's value of 3 per cent. for the "branching factor"* (*i. e.*, the fraction of the uranium atoms which follow the uranium Y actinium mode of disintegration) and 100,000 years as the period of ionium † and neglecting the correction for the difference in range between the two sets of α-particles, the period of average life of protactinium is given by the following formula in which I_{Io} and I_{Pa} represent the observed activities of the ionium and the protactinium respectively:

$$1/\lambda_{Pa} = \frac{3}{97} \times 100{,}000 \times \frac{I_{Io}}{I_{Pa}}.$$

The material used was supplied by Professor Soddy, at whose suggestion this investigation was undertaken. It consisted of a large quantity of uranyl nitrate which had been thoroughly purified in 1909 in the course of work on the radiations of uranium X ‡. The period of ionium, already cited, had been obtained by measuring the growth of radium from 6 kilograms of the purest fraction of this uranyl nitrate.

The uranyl nitrate was divided into the several fractions obtained in the last separation of uranium X in September 1909. A portion of the least pure fraction was used in preliminary tests, while the final determinations were made with the "A" fraction. The history of the preparation and the details of the chemical treatment it had undergone were thus very accurately known. It was free from thorium, and it was calculated that the amount of radium and its products which would have been formed in the twelve years since its preparation would not be sufficient to affect the measurements materially. Several kilograms at a time were used, to compensate for the comparatively short time of accumulation of the protactinium.

Methods of Measurement.

All α-ray measurements were made with a special electroscope, designed to enable a small α-radiation to be measured in the presence of an intense β-radiation. This was in order that measurements might be carried out without waiting for the decay of the uranium X which was separated with the ionium. The instrument is a modification

* *Berichte*, vol. lii. p. 1820 (1919).

† F. Soddy and Miss Hitchins, Phil. Mag. (6) vol. xxx. p. 209 (1915); F. Soddy, *ibid.* (6) vol. xxxviii. p. 483 (1919).

‡ F. Soddy and A. S. Russell, Phil. Mag. (6) vol. xviii. p. 620 (1909); F. Soddy, Phil. Mag. (6) vol. xviii. p. 858; vol. xx. p. 343.

of one described by Soddy *. Its construction is shown in figs. 1 & 2. The body of the electroscope is a brass cylinder about 10 cm. in diameter and 15 cm. in height.

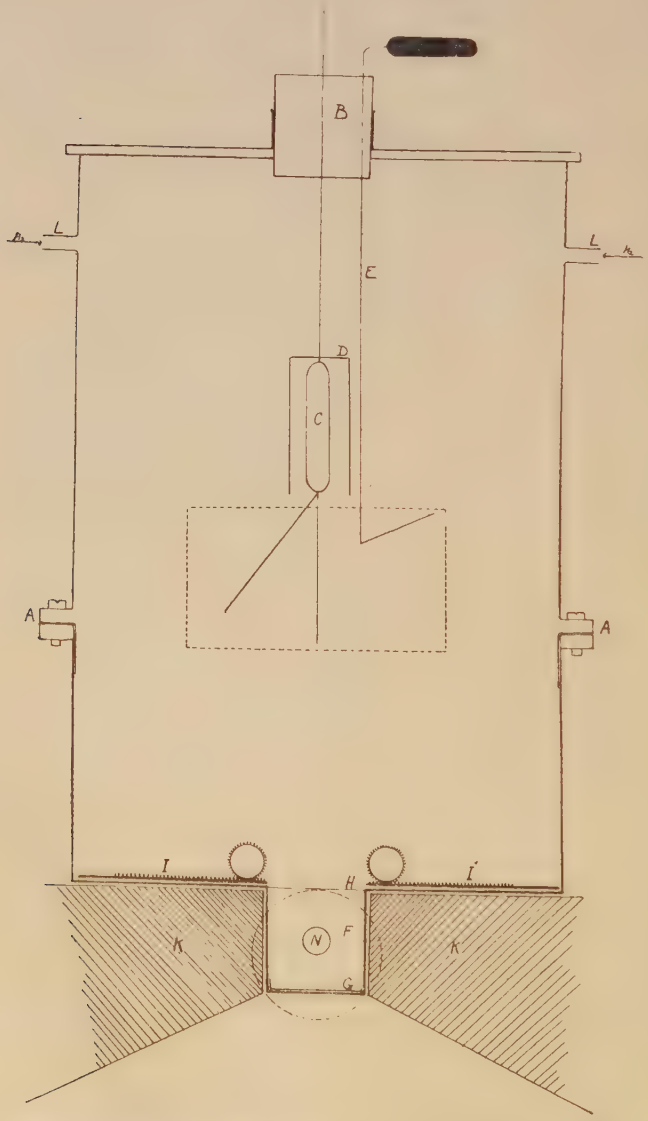
This is made in two parts, to give access to the interior if necessary. They are securely bolted together and the joint is made tight with a leather washer well coated with shellac. (The position of the flange was inadvertently placed too high in the sketch. It must come, of course, below the observation window.) A rubber cork, B, carries the ordinary gold leaf arrangement which is insulated by a sulphur bead C. D is a brass guard-ring and E a wire carrying a piece of watch-spring which may be brought in contact with the gold-leaf system to charge it. The preparation is contained in a small tray, G, about 1×6 cm., which slides into a chamber, F, fitted between the poles of a Du Bois electromagnet, with specially shaped pole-pieces K. The chamber ends in a short piece of brass tube, which is closed with a rubber stopper P. The radiation enters the electroscope through a slot, corresponding in size to the tray below it, and covered with a very thin sheet of aluminium foil H (1·4 mg. per sq. cm.). A mica screen, O, of just sufficient thickness (5·5 mg. per sq. cm.) to cut off the α-rays †, without appreciably affecting the β- and γ-rays, can be moved across the slot by means of a slide, I, operated by a rack and pinion. With the magnet turned on, the greater part of the β-radiation is prevented from entering the electroscope at all, so that the readings are of the order of magnitude of the α-radiation alone. Simple subtraction of readings taken with and without the mica screen in place give the leak due to the α-rays. The instrument is filled with hydrogen, and a current of the gas is kept passing through as the readings are made. It enters at L, passes into F through the small openings M (fig. 2), and escapes at N. This has the effect of diminishing the ionization due to the β- and γ-rays, and, at the same time, it intensifies the effect due to the α-rays, on account of the great increase of their range.

The principal difficulty in making α-ray measurements is due to the absorption of the radiation by the active material itself. The α-particles coming from the lower layers of the material are absorbed by the upper layers and consequently the effect observed does not represent the total α-radiation of the material but only that portion of it which escapes into the electroscope. To overcome this entirely, it would be

* F. Soddy, Phil. Mag. (6) vol. xviii. p. 858 (1909).
† Annual Reports of the Chem. Soc. vol. xvii. p. 235 (1920).

necessary to spread the material in an infinitely thin film. Usually an effort is made to approximate to infinite

Fig. 1.

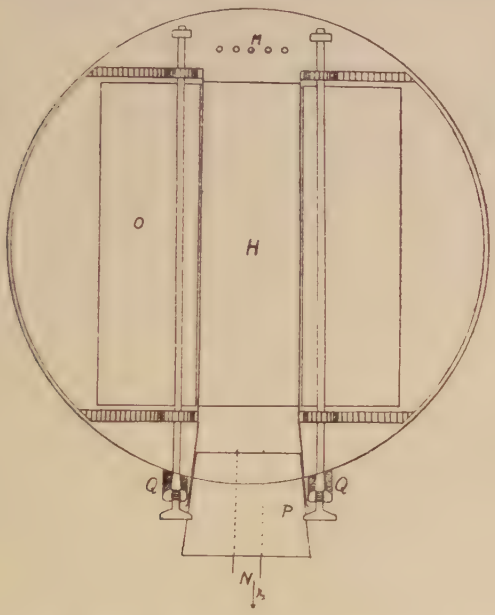


thinness by powdering the material very finely and spreading it on a metal tray or sheet with the aid of some liquid. The

necessity for using a tray of limited area in the magnet electroscope made it necessary to allow for absorption in some other way. The method adopted was based on the following considerations:—

Imagine a layer of radio-active material, one molecule thick. One half of the α-particles emitted by this layer travel downwards and are absorbed. The other half pass upward into the electroscope and, since there is no absorption by the

Fig. 2.



active material itself, the ionization observed is proportional to the number of α-particles emitted by the preparation per unit of time. Now, let a second layer be placed on top of the first. The radiation from the lower layer will be partially absorbed in the upper and the effect observed in the electroscope will be, not twice the former value, but somewhat less. As more layers of material are added, the ionization in the electroscope will gradually increase until the thickness of the material is such that the radiation from the bottom layer just fails to penetrate the surface. Further increase in the thickness of the material will cause no change in the ionization produced. This limiting value for the ionization may be called the "surface activity."

Let I_0 be the ionization which would be produced by unit

thickness of a given material if no absorption took place. Then the ionization produced by a layer of infinitesimal thickness, dw, is $I_0 dw$. Let s be the stopping-power of unit thickness of the material in terms of centimetres of air and let W be the limiting thickness which is just sufficient to stop the α-rays from the bottom layer from emerging.

Then $Ws = R$ or $W = R/s$, where R is the range in air. Let I_w be the ionization actually produced by a layer of thickness dw, at depth w below the surface :

$$\frac{I_w}{I_0 dw} = f(R, ws)\ *\ ;$$

therefore $$I_w = I_0 f(R, ws) dw.$$

The total ionization, I, produced by a thick layer, is :

$$\begin{aligned} I = \sum_{w=0}^{w=W} &= I_0 \int_0^W f(R, ws) dw \\ &= I_0 \Big[F(R, ws) \Big]_0^W \\ &= I_0 \Big[F(R, ws) \Big]_0^R \\ &= I_0 \Phi(R). \end{aligned}$$

It follows that for a given type of α-rays, for which R is a constant, the observed "surface activity" is directly proportional to the ionization which would be produced by a layer of unit thickness if there were no absorption, *i. e.*, proportional to the number of α-particles emitted by such a layer per unit of time.

Consider two separate layers of active material of weights W′ and W″ and of unit area. Let their respective "surface activities" be I′ and I″; the number of α-particles emitted per second by the whole layer N′ and N″; and the number emitted per second by a layer of unit thickness, n' and n''. It has just been shown that $n' = kI'$ and $n'' = kI''$ (k = a constant).

$$N' = W' n' = W' k I',$$
$$N'' = W'' n'' = W'' k I''; \qquad \text{therefore} \qquad \frac{N'}{N''} = \frac{W'I'}{W''I''}.$$

This was the principle of the method of measurement

* Rutherford, 'Radio-active Substances and their Radiations,' p. 160. Rutherford states: "... the total ionization due to thick layers of radio-active matter of different kinds is approximately proportional to R^2, where R is the range of the α-particles emitted."

employed. The preparations were of more than the limiting thickness, so that the activity measured in each case was the "surface activity." This, in arbitrary units (the rate of leak of the electroscope in scale divisions per minute), was multiplied by the weight of the preparation and the figure so obtained was taken as proportional to the total activity of the material, *i. e.*, the entire number of α-particles given out per second. An effort was made to have the preparations which were compared as nearly alike as possible, both in weight and composition.

The Chemical Separations.

As ionium is isotopic with uranium X, any chemical method of separating the one applies equally to the other. Owing to the high degree of purity of the uranyl nitrate many of the ordinary methods for separating uranium X failed. The method finally adopted was as follows:—The uranyl nitrate was treated by extraction with ether *. The final water layer was evaporated to dryness, ignited carefully to uranic oxide, UO_3, and dissolved in hydrofluoric acid. A very small amount of insoluble residue contained the greater part of the ionium and uranium X. (A satisfactory separation by simple precipitation with hydrofluoric acid could not be obtained. The fluorides of cerium, thorium, etc., are quite soluble in solutions of uranium salts such as uranyl nitrate. The ordinary black oxide of uranium, U_3O_8, refuses to go into solution in hydrofluoric acid, but uranic oxide, UO_3, dissolves quite readily, leaving the insoluble fluorides behind.) The separation of ionium by this method was not absolutely complete but its extent could be determined by β-ray measurements of the uranium X. The exact proportion of the total ionium present in any preparation could in this way be readily calculated.

When the uranyl nitrate was first brought into solution, it was found to contain a certain amount of dust and dirt. It was thought possible that this might have carried down with it some of the protactinium, which may be expected to share the excessive insolubility of its analogue, tantalum. The dust was filtered out and boiled with hydrofluoric acid, the extract being added to the above hydrofluoric acid solution. The latter was then poured into ammonium carbonate solution to get rid of the uranium. To the resulting precipitate, after ignition and boiling with nitric acid, was added the residue

* Soddy, 'The Chemistry of the Radio-Elements,' p. 38.

from the extraction of the dust. The whole was then spread on a platinum tray and its α-radiation measured. It contained the whole of the protactinium present, with a small amount of ionium which was estimated from the isotopic uranium X present by β-ray measurements.

The fluoride precipitate containing the greater part of the ionium was then added to it, the two were thoroughly mixed and the α-ray measurements repeated. The expedient of mixing the two, rather than measuring them separately, was adopted on account of the very small amount of the fluoride residue, in order that the preparations compared should be as nearly as possible of the same composition.

The method of calculation is illustrated by the following, which gives the result of this determination :—

Weight of uranyl nitrate used:		4·780 kgm.
	Pa.	Pa+Io.
Weight of preparation	0·0804	0·0912 gm.
α-activity	5·55	15·78
β-activity	9·03	70·91

β-ray measurements showed the fluoride residue to contain 40 per cent. of the original ionium present in the uranium.

β-activity of (40+X) per cent.	Io	=	70·91
,, X per cent.	Io	=	9·03
,, 40 per cent.	Io	=	61·88

therefore $X = 9{\cdot}03/61{\cdot}88 \times 40 = 5{\cdot}8$ per cent.

α-activity of 45·8 per cent.	Io+Pa	$= 15{\cdot}78 \times 0{\cdot}0912 = 1{\cdot}4391$
,, 5·8 per cent.	Io+Pa	$= 5{\cdot}55 \times 0{\cdot}0804 = {\cdot}4662$
,, 40 per cent.	Io	$= {\cdot}9729$
,, 5·8 per cent.	Io	$= {\cdot}1431$
,,	total Io	$= 2{\cdot}4322$
,,	Pa = ·4662 − ·1431	$= {\cdot}3231$

$$\text{therefore } 1/\lambda_{Pa} = \frac{3}{97} \times 100{,}000 \times \frac{2{\cdot}4322}{{\cdot}3231} = 23{,}300 \text{ years.}$$

A previous determination was carried out in a similar manner but the activity of the residue from the hydrofluoric acid extraction of the dust was measured separately. The result obtained was 15,500 years.

In the preliminary experiments on "D" fraction of the material, the chemical methods used were quite different. After repeated fractional crystallization of the uranyl nitrate, the mother liquor was poured into dilute ammonium carbonate

solution. The precipitate carries down with it the radio-active elements present in minute quantity in the uranium *. This process of crystallization was repeated several times till the last precipitate showed a negligible α-activity. After α-ray measurements on the total ammonium carbonate precipitate (Pa + Io), it was dissolved in nitric acid, and the protactinium removed by shaking with successive small portions of tantalic acid † until the last of these showed a negligible α-activity. These were then combined and their total activity measured.

The first rough trial on one kilogram of material gave a figure of 28,800 years for the period : a second and more careful application of the method on two kilograms of material gave 16,000 years.

The same method was then applied to a portion of the "A" fraction of the uranyl nitrate. This was so free from impurities that only a faint precipitate was obtained with ammonium carbonate, which gave only a very slight separation of the ionium. The ether and fluoride separation, already described, was then applied. In this case, the ionium fluoride was not combined with the protactinium preparation but was measured separately, a little tantalic acid being added to it to make the weights of the two preparations approximately alike. The period was calculated to be 17,300 years.

In all cases, the behaviour of the ionium at all stages of the treatment could be followed and the proportion present in any preparation ascertained by measurements of the β- or γ-radiation of its isotope, uranium X. Efforts were directed, therefore, to obtaining an accurately known portion of the ionium present for measurement, rather than the whole. Unfortunately, no similar means of tracing the protactinium were available. It was assumed to follow the behaviour which might be expected of it as an analogue of tantalum. Evidence that this is justified is provided by the fact that the result of the second experiment is in good agreement with the later ones, and in this earlier method the completeness of the separation of the protactinium and the ionium from the uranium and from each other was tested by repeating the process of separation and testing each successive product until it ceased to give a measurable α-radiation. If the removal of the protactinium was in any

* Soddy, 'The Chemistry of the Radio-Elements,' p. 38.

† Hahn and Meitner, *Phys. Zeit.* vol. xiv. p. 758 (1913).

case incomplete, then the value calculated for the period is a maximum one.

In the calculations, the difference in range between the α-particles of ionium and protactinium has not been taken into account. The ranges are close together (3·1 and 3·5 cm.) so that the correction would in any case be small, probably not larger than the experimental error.

Results.

The results obtained may be tabulated as follows :—

Material.	Weight (in kgs.).	Chemical treatment.	Average Life Period of Pa.
"D"	1	Crystallization and ammonium carbonate.	(28,200) yrs.
"D"	2	Do.	16,000 „
"A"	2·5	Crystallization and ammonium carbonate followed by ether and fluoride.	17,300 „
"A"	4·197	Ether and fluoride.	15,500 „
"A"	4·780	Do.	23,300 „
		Mean	18,000 yrs.

The agreement among the individual values is reasonably satisfactory, especially in view of the very considerable alterations in the procedure between the separate determinations. Neglecting the first, which was merely a rough trial of the method, the mean is 18,000 years, which corresponds to a half-value period of 12,500 years. This closely confirms Hahn and Meitner's conclusion.

The period was also calculated by comparing the activity of the protactinium with that of uranium. The formula used was $1/\lambda_{Pa} = \frac{\frac{1}{2}I_u}{I_{Pa}} \times 0{\cdot}03t$, where t is the time from purification in years.

The values used for I_u and I_{Pa} were, of course, weight times activity; in the case of the uranium, the weight taken being the weight of uranium oxide equivalent to the entire amount of uranyl nitrate from which the protactinium had been separated.

The results obtained were :—23,700 ; 26,400 ; 9,500 ; 20,800. Neglecting the third, which for some reason displays a wide discrepancy from the others, the mean is 23,600, which corresponds to a half-value period of 16,400.

Exactly the same method of calculation may be used to arrive at the period of ionium. The results found were :— 149,000 ; 153,000 ; 62,000 ; 79,200. While the mean of these is not far from 100,000, they differ too widely among themselves for any reliance to be placed upon them. This fact casts some doubt on the reliability of the calculations of the period of protactinium from comparison with uranium, and the value obtained by the comparison with ionium is probably more certain.

The foregoing is based on the usual assumption that protactinium is the product of the short-lived uranium Y. If, as Dr. A. S. Russell has suggested in a recent letter to 'Nature' *, the parent of protactinium is not uranium Y, but an unknown element of comparatively long period, the value here found for the period of protactinium is a maximum, and must be reduced to an extent depending on the period of its parent.

In conclusion, I wish to express my very grateful acknowledgement to Professor Soddy for his invaluable advice and suggestions, and for supplying the unique uranium preparation which made this work possible.

Summary.

The period of protactinium has been estimated from a comparison of the α-activities of the ionium and protactinium separated from a large quantity of uranium purified in 1909 by Professor Soddy. The mean value obtained is 18,000 years for the period of average life, which is in substantial agreement with the period of half-change, 12,500 years, obtained by Hahn and Meitner.

Old Chemistry Department,
University Museum, Oxford.

* 'Nature,' vol. iii. p. 703 (May 26th, 1923).

LXXVII. *Forced Oscillations in Self-maintained Oscillating Circuits. (Thesis accepted for the Degree of Ph.D. in the University of London* *.) *By* W. A. Leyshon, *B.Sc.* †

Contents.

Section 1. Introductory.
,, 2. Apparatus.
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Section 1.—*Introductory.*

THE present paper is concerned with the behaviour of certain electrical circuits maintained in oscillation by a three-electrode thermionic tube, or triode, under the influence of impressed oscillations.

If two inductance-capacity valve-maintained oscillating circuits are closely coupled, and in tune, it is well known that the capacity of the condenser in either circuit can be varied within comparatively wide limits without the phenomenon of beats occurring. The circuits seem to pull each other into tune, even when their proper frequencies (if uncoupled) differ considerably. The extent to which this "pulling into tune" occurs increases with the coupling.

Some experiments by Vincent suggested that the phase difference between the oscillating currents in the two circuits changed from 0° to 180° (or *vice versa*) as the capacity of the condenser in one circuit was varied. Two circuits which are mutually held in this way at the same frequency, although not necessarily in phase, may be said to be in the "accordant state."

In the experiments to be described an alternating voltage was applied to an oscillating triode system, either by means

* A paper by Dr. E. V. Appleton on "The Automatic Synchronization of Triode Oscillators" appeared in the Proceedings of the Cambridge Philosophical Society, vol. xxi. pt. 3, Nov. 14, 1922, at the moment the writing of this thesis was approaching completion. Dr. Appleton attacks a problem similar to that of the present writer by an entirely different method.

† Communicated by Prof. W. H. Eccles, F.R.S.

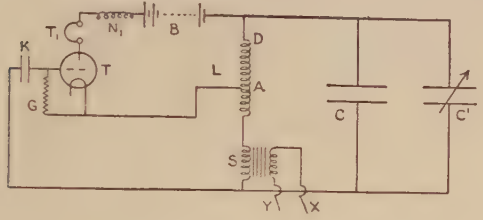
of a second triode system, which was itself acted on by the first system, or by means of a circuit (possessing constant amplitude and frequency) maintained by a tuning-fork and two triodes in parallel.

Low frequency oscillating circuits were employed and the phase differences it was desired to observe were shown by means of a Lissajous instrument designed for the purpose which is described in Section 3.

Section 2.—*Apparatus.*

Experiments were made with circuits arranged as in fig. 1

Fig. 1.



N_1 = coils of Lissajous' instrument.
T_1 = telephones.
T = triode ("R" pattern).
K = grid condenser (·02 microfarad).
G = grid leak (ca. 1 megohm).
L = inductance.
C = bank of condensers or fixed condenser.
C′ = variable air-condenser.
S = coupling coil.
X, Y = points connected to second oscillatory circuit.

In fig. 1 the condensers C and C′ are shown in shunt to the whole inductance L and coupling coil S. In some experiments they were in parallel with DA only.

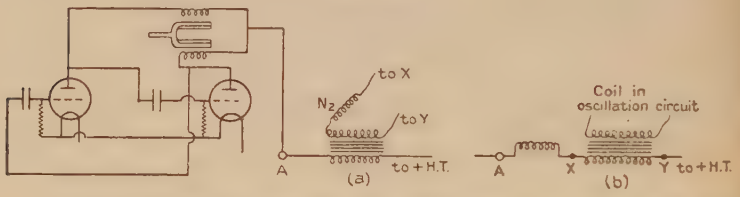
In some experiments S consisted of two three-layered solenoids which could be adjusted in position on a common iron core ("weak" coupling); in others, of two or four "biscuit coils" wound on a laminated stalloy core, and having a much greater mutual inductance than that of the solenoids ("strong" coupling).

The points X, Y were connected either to a circuit similar to that shown in fig. 1, or to a circuit in which oscillations of constant frequency and amplitude were maintained by

means of a tuning-fork set *. This is shown in fig. 2, in which (*a*) shows the circuit coupled to the tuning-fork set in which X, Y are connected, and (*b*) shows one coil of S included in the battery circuit of the tuning-fork set.

Two tuning-forks were used, one having a frequency of 450, and the other of 256.

Fig. 2.



The inductance-capacity circuits were tuned, first by adjusting the capacity C (fig. 1), then by adjusting the filament temperature of the triode, then by adjusting the voltage applied to the triode, and finally by means of the variable air-condensers in parallel with C.

In the experiments to be described, the circuits were usually so coupled that the accordant state persisted over a limited range of alteration of the capacity of C" (fig. 1).

The phase relations between corresponding currents in the two coupled circuits were observed by means of the instrument described in the next section.

Section 3.—*The Lissajous instrument.*

This consisted of a very fine magnetized steel needle, with the eye removed, which was kept in vibration by means of two perpendicular pairs of coils, through which the respective alternating currents were passed. The vibrations of the needle were observed by means of a microscope.

The needles used were "superfine beading" needles, which are longer and finer than the finest sewing needles manufactured. The average diameter was about 14 mils. In cutting the needle to the desired length, the inner crystalline structure was exposed, so that when the light of an electric lamp was focussed on to the cut surface, a number of brilliant points of light were seen through the microscope. The figures described by these points of light could be very clearly seen in the course of the experiments. The needle used for any particular experiment had a natural frequency

* Eccles & Jordan, "Sustaining the Vibration of a Tuning-fork by a Triode Valve," Electrician, June 20, 1919. "A Method of using two Triode Valves in parallel for generating Oscillations" (Paper read before B.A. (Eng.) Meeting, 1919).

not very different from that of the electrical circuits whose oscillations caused it to be set in vibration.

In the first form of the apparatus, the point of the needle was pushed into a cylinder of wood which fitted into a vertical hole in a block of wood placed on the stage of the microscope. Horizontal grooves in this block carried the exciting coils, whose positions could be adjusted slightly by means of small wedges placed on either side of each coil and between the coil and the walls of the groove.

In a later form, the wooden cylinder carrying the needle was a short piece of hard wood (actually a piece of a penholder) which fitted tightly into a brass tube. The brass tube could be screwed through a hole in a brass plate, and was kept in position by a nut.

The brass plate, which rested on the stage of the microscope, supported by means of three pillars a shallow square brass box, into which a square block of wood fitted.

The exciting coils were fitted in horizontal grooves in this block, their positions being slightly adjustable as before. Figs. 3 and 4 show the arrangement of the apparatus, fig. 4 showing the needle in position in the holder.

The exciting coils were Brown 4000 ohm A type telephone coils, which had been modified as suggested in figs. 5 (*a*) and (*b*). The pole-pieces AB were filed down to

Fig. 3.

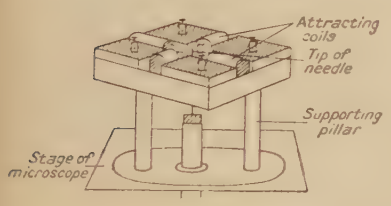


Fig. 4.

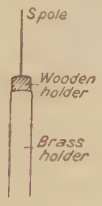


Fig. 5.

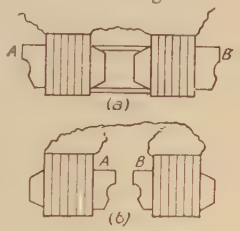
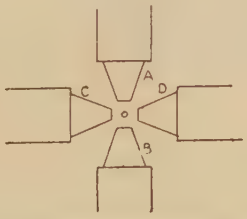


Fig. 6.



a thickness of 1 mm. at the ends, so that they could be brought within a small distance of the upper end of the needle.

Fig. 6 shows the arrangement of the exciting coils and needle as seen from above.

The eyepiece of the microscope finally used was fitted with a micrometer marked out in $\frac{1}{4}$-mm. squares, so that it could readily be verified that the straight lines seen when the coils AB alone were in action were in the direction AB, and accurately perpendicular to the straight lines seen when the coils CD were in action alone.

With both coils in action and currents of the same frequency passing through them, the Lissajous figure corresponding to the difference of phase between the currents was seen.

When the currents differed in frequency, the changing figures during the beats could be observed with ease.

It was found advisable not to work too near the tune of the needle. At this point the amplitude of oscillation increased to a great extent, and if one set of coils only were in action, the figures seen as the exciting current passed through the natural frequency of the needle were as shown in fig. 7.

Fig. 7.

The figures are probably due to the difference in proper frequency of the needle in the two perpendicular directions, a small oscillation in the $\backslash$ direction being maintained near the tuning point. It should be noted that the changes (a) to (e) occurred for quite small variations in frequency, particularly when a needle of almost circular section (as seen under a microscope magnifying a hundred times) was used.

The following considerations also indicate that it is better to use a mistuned needle :—

Suppose the amplitude of oscillation small (which it was in all the cases considered).

Let p_1 = period of (undamped) needle in one direction.

Let p_2 = period of (undamped) needle in the perpendicular direction.

Then
$$\tan \delta_1 = \frac{2k_1 n}{{p_1}^2 - n^2},$$

where k = damping factor,

n = frequency of impressed force,

δ_1 = phase angle between displacement and impressed force.

Similarly $$\tan \delta_2 = \frac{2k_2 n}{p_2^2 - n^2}.$$

Provided $k_1 \simeq k_2$ and n differs considerably from p_1 and p_2, the needle shows very approximately the phase difference between the impressed forces.

But if n is very nearly equal to p_1 and $p_1 \neq p_2$, there may be a considerable error due to the difference between δ_1 and δ_2.

To calculate the length of the needle required for a particular frequency, the formula $N_1 = \frac{0{\cdot}525kb}{l^2}$* may be used,

where N_1 = frequency,

k = radius of gyration of cross-section about axis $\left(\frac{r}{2}\right.$ for cylindrical rod radius $\left.r\right)$,

q = Young's modulus,

ρ = density,

l = length,

$b = \sqrt{\frac{q}{\rho}}$.

If N = (say) 50,

ρ = 7·8,

q = $20{\cdot}9 \times 10^{11}$, ; this gives $\frac{r}{l^2} = 0{\cdot}000368$.

For the needles used, $r \simeq 0{\cdot}015$;

$$\therefore \quad l = 5\sqrt{\frac{0{\cdot}015}{0{\cdot}000368}},$$

$$= 6{\cdot}4 \text{ cm.}$$

In an actual experiment it was found that for $l = 5$ cm., $N \simeq 85$.

Therefore for $N = 50$,

$$l = \sqrt{\frac{85}{50}},$$

$$= 6{\cdot}5 \text{ cm.}$$

* Barton, 'Theory of Sound,' p. 292.

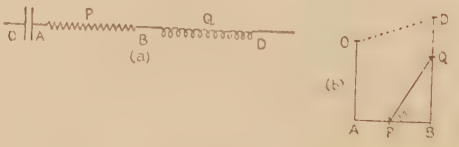
Section 4.—*Experiments to test the Lissajous instrument as an indicator of phase difference between the currents in the pairs of coils.*

Suppose an alternating current sent through a circuit consisting of a condenser, a resistance, and an inductance in series *.

The variations of the P.D. between O and D with time can be represented in the well-known way by an Argand diagram. If the variations in the P.D. between O and A and between B and D with time are represented similarly in the same diagram, the radius vector representing the P.D. between O and A will be 90° behind that representing P.D. between A and B, and that representing the P.D. between B and D will be 90° ahead of the resistance radius vector.

The direction in the Argand diagram of the radius vector representing the P.D. between O and D is found, as shown in fig. 8, by the dotted line OD.

Fig. 8.



Similarly the line PQ gives the direction and length of the radius vector representing the P.D. between the points P and Q, the direction of the resistance radius vector being given by AB.

If the points O and A are connected by a resistance of such value that the current flowing in this shunt circuit is negligible in comparison with that flowing through the main circuit, then this current will be in phase with the P.D. across the condenser, and therefore 90° out of phase with the current in OABD.

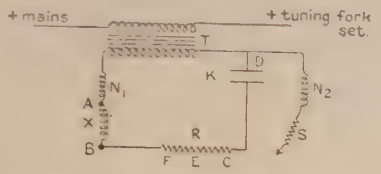
As indicated in fig. 8 (*b*), the current in such a shunt across PQ would make an angle of ϕ with the main current.

The apparatus used in the tests to be described is indicated in fig. 9.

* Eccles, 'Continuous Wave Wireless Telegraphy,' p. 139.

The results obtained are given in Table I. It should be noted that the impedance of the coils N_1, N_2 is practically resistive at frequency 256.

Fig. 9.



T = iron-cored transformer, the primary of which was included in the battery circuit of the T.F. set.

K = condenser.

L = inductance.

R = resistance.

N_1, N_2 = coils of Lissajous instrument.

S = resistance in series with N_2.

In some experiments N_1 was placed in shunt to some part of the circuit.

TABLE I.

	Position of N_1.	Position of N_2.	Figures seen.	Notes.
1.	In the oscillation circuit.	DC	O, provided condenser was sufficiently large.	
2.	"	In ‖ with N_1.	\ [in phase].	Using a needle more nearly tuned to 256, figure seen was ◊.
3.	Across DC with about 10,000 ohms in series.	AX	/ [out of phase].	
4.	In the oscillation circuit.	AX	O	
5.	BF	DE	O→◊	As part of R in series with condenser was increased.

From these and similar experiments, it was concluded that a distuned needle could be relied on as an indicator of phase difference.

SECTION 5.—*Experimental results obtained with two coupled oscillating circuits.*

1. Lissajous' figures seen in the accordant state.

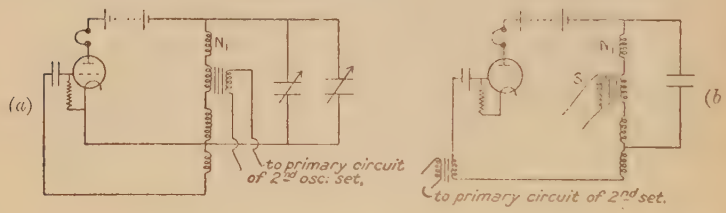
(*a*) Two inductance-capacity oscillating sets.

Some typical results are shown in Table II.

As already stated, the first experiments made were with the apparatus of fig. 1.

Other experiments have been made with the circuits shown in fig. 10.

Fig. 10.



In making the first sets of observations (1 in Table II.), it was observed that the same figures were obtained in the same order when, instead of increasing the capacity of the variable condenser in Set 1, the capacity of the variable condenser in Set 2 was diminished.

It was also observed that with sufficiently strong coupling between the circuits, the same figure was seen for the same setting of the variable condenser, whether the setting were approached from a larger or a smaller value.

This was not the case with weaker coupling—the same figure was seen for a higher condenser reading when the accordant state was reached by increasing the capacity of the tuning condenser than when it was reached by diminishing the capacity.

It was found on diminishing the coupling between the oscillating circuits, these being otherwise left unchanged, that provided the change in coupling were not too great, the same figures were seen, but the range of capacity over which the accordant state was observable diminished with the coupling.

The observations referred to in 1 (Table II.) were made with different needles—to suit the different frequencies employed.

TABLE II.

	Circuits.	Coupling.	Figures seen.	Frequency.	Notes.
1.	As fig. 1.	Strong or weak.	Set 1 condenser increasing ⟶ ⟋○⟍	85–450	In these first observations it is not certain which is the in-phase figure.
2.	As fig. 1, but needle coils in oscillation circuit.	Strong −	⟋○⟍0	ca. 256	⟍ is the in-phase condition.
3.	As fig. 1.	Strong.	0○⟍0	,,	,,
4.	Fig. 10(*a*).	Weak.	⟍○⟋	,,	Apparently, but very small range of accordant state. Both sets oscillating strongly.
5.	,,	Fairly strong +	0⟋○0	,,	⟍ is the in-phase condition.
6.	,,	,, −	0⟍○0	,,	,,
7.	,,	Strong + ,, −	0 0	,,	This figure was seen over whole range of capacity of tuning condenser.
8.	Fig. 10(*b*).	,,	○⟋○	,,	⟍ is the in-phase condition.

Those referred to in 2–7 were made with one of the needles which were fitted in turn in the apparatus when this was tested as a measurer of phase difference.

With conditions as in 3, Table II., and the figure *0* showing, stopping Set 2 (by breaking the filament or battery circuit) caused the note of Set 1 to be raised appreciably in pitch.

With conditions as in 7, Table II., and figure ○ (Set 1 condenser large), stopping Set 2 (as above) lowered the frequency of Set 1, and stopping Set 1 raised the frequency of Set 2.

The accordant state figures have been observed, using one set an octave above the other*; the tuning was very sharp, even with strong coupling. This case has not been further investigated.

* *Cf.* "Sur la synchronisation harmonique des oscillations électriques." Mercier, *Comptes Rendus*, clxxiv. pp. 448–450, Feb. 13, 1922.

It should be observed that in all the arrangements used (except that of fig. 10, *a*), each oscillating set caused a straight-line oscillation of the needle between the coils connected in that set when the filament or anode battery circuit of the other set was broken.

In the case of the arrangement used in fig. 10 (*a*), this was not so.

Accordant state figures have been seen using the circuit of fig. 1, except that the grids of the two oscillating sets were connected through a condenser, no coupling transformer being used in the circuits.

1 (*b*). Lissajous' figures seen in the accordant state—using a tuning-fork set and an inductance-capacity set :—

The results given in Table III. were obtained, using the circuits of fig. 2 (*a*) and (*b*), and of figs. 1 and 10 (*b*) (coupling coils S omitted).

TABLE III.

	Circuits.	Coupling.	Figures seen.	Frequency.	Notes.
1.	Figs. 1 & 2 (*b*).	Strong. T.F. set oscillating strongly.	Set 1 condenser increasing — → O ⁄	450 & 256	Not known which is the in-phase condition.
2.	"	Strong. Inductance capacity set oscillating very strongly—or amplitude of T.F. set not very great.	\ O jerked to ⁄	"	. .
3.	Figs. 10(*b*) & 3(*a*).	Fairly strong +	O \ jerked to O *0*	...	...
		" —	O jerk *0* ⁄ O		
4.	..	Strong +	∽ \ O	...	\ in-phase..
5.	"	" —	O ⁄	...	...

+ coupling in the above Table means that when the currents in the circuit containing the primary of the coupling coil, and in the oscillating circuit were in phase, the grid voltage and what may be termed the "according voltage" were also in phase.

Under conditions 1 and 2 of Table III., it was found that the range of capacity of the tuning condenser covering the accordant state was diminished with diminished amplitude of the tuning-fork set.

With "weak" coupling very complicated figures were

sometimes obtained, one figure changing abruptly to another as the tuning condenser was varied steadily.

With a crystal detector and galvanometer across this tuning condenser (and a high resistance in series with these), the sudden changes of figure were seen to be accompanied by sudden slight changes in the current through the galvanometer.

With conditions as in 4, Table III., breaking the filament circuit of the tuning-fork set so as to stop the vibration of the fork (figure ○ showing) lowered the frequency of Set 1. With the figure ⊂ showing, breaking the filament circuit of the tuning-fork set raised the frequency of Set 1 slightly.

With conditions as in 5, Table III., breaking the T.F. set filament circuit lowered the frequency when figure *0* was showing.

With conditions as above, reducing the amplitude of oscillation of the tuning-fork circuit by lowering the filament current caused the figure to change to ○, and finally to \ just before the oscillations of the T.F. set died away. (This apparently corresponds to 2, Table III.)

2. Lissajous' figures observed during beats :—

Provided that the coupling between the two oscillating circuits was not too strong, so that the beat frequency was not too great when the accordant state had just been upset, it was possible to watch the phase changes during the beating process.

These phase changes were found to occur uniformly with time (*i. e.* the change from one figure to another, representing a change of phase difference ϕ, took the same fraction of a whole beat period whatever the first figure observed) when there was no coupling between the circuits. It was found impossible, with the tuning condensers used, to bring the circuits exactly into tune with this arrangement.

With coupling between the oscillating circuits, the phase changes did not occur uniformly with time during the beats. Using the circuits and conditions of Tables II. and III. (1), it was observed that the non-uniform phase changes during beats were different on the two sides of the accordant state—*i. e.*, for the higher and lower settings of the tuning condenser.

It was observed that the figure last seen in the accordant state before beating began was the figure longest in view during the beat.

With loose coupling between the tuning-fork set and an inductance-capacity set (when the figures changed by jerks

in the accordant state), the figures seemed to go through a kind of drill when beating began.

Increasing the coupling between the two oscillating circuits intensified the non-uniformity of the beats; with sufficiently strong coupling between the circuits, changes of frequency corresponding to the changes of figure could be heard in the telephones in the oscillating circuit.

Section 6.—*Notes on the results obtained.*

The foregoing experiments on the accordant state suggest generally that when an alternating voltage of constant frequency and amplitude is applied to the grid of an oscillating triode, this voltage not differing markedly in frequency from that of the undisturbed oscillating system, the latter may be pulled into tune with the according voltage if the latter is sufficiently great. The phase difference between the grid voltage as supplied by the oscillating triode and the according voltage in the accordant state depends on the strength of the latter, and on the difference in the frequency of the system supplying the according voltage and the undisturbed frequency of the triode system.

When the according voltage is not sufficiently strong to pull the triode system into tune, modified beats are observed, the rate of change of phase with time being least when the phase difference is such that the triode system is most nearly in tune with the applied voltage.

In the experimental results obtained, only one-quarter of a cycle of phase change has been observed in the accordant state with comparatively strong according voltages.

When an alternating voltage supplied by a triode system is applied to a second triode system, which itself affects the first system, half a cycle of phase change has been observed to take place between corresponding currents in the two triode systems.

In conclusion, my thanks are due to Professor W. H. Eccles, F.R.S., for suggesting the problem and the method of investigating it, for the loan of much apparatus, and for his help and advice; also to Dr. J. G. Barns for the use of his microscope; and to my father, Mr. R. Leyshon, for his help in developing the Lissajous instrument during the summer vacation.

November 1922.

LXXVIII. *A Statistical Investigation of the Visibility of Red Light.* *By* R. A. Houstoun, *Ph.D., D.Sc., Lecturer on Physical Optics in the University of Glasgow, and* Eric W. M. Heddle, *M.A., B.Sc., Lecturer in Mathematics in the Royal Technical College, Glasgow* *.

DURING the past six years or so a statistical survey of the colour vision of some two thousand students has been carried out at the University of Glasgow. This survey has shown that some observers have better colour vision than the average and some worse, following the law of normal deviation, and that normal deviation almost covers everything so far as women are concerned. In the case of men, however, the colour blind and red and green anomalies occur scattered outside the normal curve.

Some observers see the red end of the spectrum much dimmer than others. The object of the present investigation was to see whether these observers were an extreme case of normal variation or not, and also to find to what extent they occurred. The investigation has also a bearing on the original Young-Helmholtz theory of colour vision, according to which the colour blind were divided into two well-defined classes, the red blind and the green blind, a view which Helmholtz himself afterwards abandoned.

§ 1. We find the most valuable previous data on the subject in a paper by Coblentz and Emerson †. By means of a flicker photometer they determined the relative visibility of the different colours of the spectrum for 125 different observers who were apparently a random distribution so far as colour vision is concerned; their results are exhibited in a table. The wave-length 665 $\mu\mu$ is the point nearest the red end of the spectrum for which every one of the 125 observers made a reading. The table gives the ratio of the visibility of this colour to the visibility of the most visible colour in the spectrum for each of the observers. The results vary from ·252 to ·822. The most visible colour in the spectrum is always in the green, though its position varies somewhat from one observer to another. The two numbers cited above, therefore, give the ratio of the brightness of the red to the brightness of the green, multiplied by a constant factor, for the two observers in question. If the logarithm of this ratio is taken, the 125 observers can be

* Communicated by the Authors.

† "Relative Sensibility of the Average Eye to Light of Different Colours and Some Practical Applications to Radiation Problems," Bull. Bur. Standards, xiv. p. 167 (1918).

grouped in the following manner in terms of its value; in the table the characteristic of the logarithm is omitted :—

Logarithm.	No. of Observers.	Logarithm.	No. of Observers.
·40 & < ·44	2	·68 & < ·72	19
·44 & < ·48	0	·72 & < ·76	18
·48 & < ·52	3	·76 & < ·80	11
·52 & < ·56	8	·80 & < ·84	7
·56 & < ·60	10	·84 & < ·88	2
·60 & < ·64	18	·88 & < ·92	1
·64 & < ·68	26		

The table is plotted in fig. 2. The number of observers is grouped against logarithm of ratio instead of against ratio for a reason that will be explained later in connexion with our own results. The distribution represented in fig. 2 is not far off the normal curve. Coblentz and Emerson subdivide their observers into various groups for convenience of handling, red sensitive, blue sensitive, &c., but these groups are arbitrary. There is nothing in the paper to show that we are not dealing with one homogeneous population.

§ 2. As we desired only to get one reading for each observer, the arrangement of Coblentz and Emerson would have been too elaborate for our purpose. We made three false starts before obtaining an apparatus to our taste.

(1) First of all an attempt was made with a specially-fitted spectroscope to put the cross-wire on the red end of a continuous spectrum. We had never much hope of this method, and it failed owing to diffuse light in the instrument and variation of the reading with the state of dark adaptation of the observer's eye.

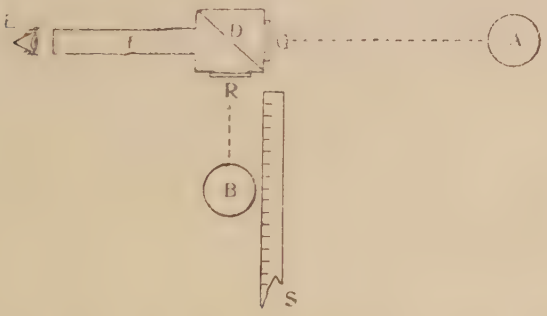
(2) A thin piece of copper with two very fine holes drilled in it was placed on the stage of a low-power microscope. Behind one of these holes was a piece of red glass; behind the other a wedge of neutral glass. On looking into the microscope the observer saw a red disk, something like a signal lamp at a hundred yards or so, and a grey disk beside it. By rotating a wheel which carried the wedge he adjusted the luminosity of the grey disk, until the two disks were equally bright. This method was abandoned, because the

"neutral" wedge had a green tint, because the range of adjustment was not great enough, and because a slight displacement of the piece of copper on which the holes were drilled had a great influence on the result.

(3) We next measured a red light against a white one by means of a Simmance–Abady flicker photometer. But the fact that this instrument gives more consistent readings than the grease-spot or shadow photometers when the colours are different was outweighed by its inconvenience and unfamiliarity to the observers. So it was abandoned in favour of the simple arrangement of the grease-spot type the plan of which is shown in fig. 1.

T is a tube about 18 cm. long and of 2·5 cm. diameter, with a small hole at the end of 4 mm. diameter into which the eye E of the observer looks. He sees a diamond shaped

Fig. 1.



opening D of 8 mm. side in a piece of paper which is stretched diagonally across a cubical box of 5 cm. side. Through this opening a ground-glass plate G is visible; this plate is illuminated by an 8 candle power carbon glow-lamp A at a fixed distance of 2 metres. Light from a 16 candle-power carbon glow-lamp B falls on the front of the paper through the piece of clear red glass R. Consequently the observer sees a white diamond against a red background. The lamp B moves along a scale S 50 cm. long, and its position on this scale is adjusted so that the background has the same intensity as the diamond. The same lamps were used throughout the whole series of tests; they were mounted at about a height of 30 cm. above the table. There were screens, not shown in the diagram, to prevent direct light from the lamps reaching the eye of the observer. The red glass was a pure red of the kind used

for the signal lamps; it transmits from the red end of the spectrum to about 610 $\mu\mu$, and is much purer than Wratten and Wainwright's standard tricolour red, which contains some yellow.

The tests were carried out between April 1922 and June 1923, partly in the Natural Philosophy Department of the University and partly in the Natural Philosophy Department of the Technical College. Altogether 604 students were examined, 56 women and 548 men. Of these 373 were University students, arts, pure science, engineers, and medicals, and 129 were day and 102 evening students at the Technical College. They made the test during the optical laboratory course, simply leaving the optical bench or whatever it was they were working at and returning to it when the test was completed. The distribution was essentially a random one; the observers were not selected in any way, but simply taken as they came, and no one who was asked to make the test declined.

In order to have conditions exactly the same in each case instructions were not given orally, but by means of a written card, which stated that "the light on the right-hand side was to be moved backwards and forwards along the metre stick until the white diamond and the surrounding red appeared of the same brightness." Each observer made three settings; the first of these was intended only to give familiarity with the apparatus and was consequently not recorded, but the second and third were recorded and their mean taken. The apparatus, it should be stated, worked very satisfactorily; it was simple and straightforward, and all the observers understood at once what was wanted.

Most observers, of course, find it extremely difficult to say when a white surface and a red one are equally bright. Helmholtz himself had no confidence in his judgment on this point. But it is easy to arrange that first the red, and next the white, is the darker of the two and then leave the lamp midway between these two positions, and this is how the observers usually proceeded.

As the scale was calibrated by an auxiliary experiment, it was possible from each observer's settings to state what fraction of the total light of the glow-lamp the red glass in question transmitted for him. The results varied from ·086 for the man who was most red sensitive to ·0015 for the man who was least red sensitive. This fraction specifies the observer's red sensitiveness. Its reciprocal specifies red blindness. Now in drawing the frequency curve the shape differs according as we use the fraction or its reciprocal as

abscissa. In the one case the left-hand side will be crowded together and the right-hand side extended, while in the other case the opposite will happen. If, however, we were to use the logarithm of the fraction or of its reciprocal as abscissa, the shape of the curve would be the same in each case. As there is no particular reason for preferring the ratio to its reciprocal, or *vice versâ*, we have stated the results in terms of the logarithm, and, in order to avoid negative characteristics, have chosen the logarithm of the reciprocal, or rather, for convenience in calculation, the logarithm of a number proportional to the reciprocal. The results are given in the following table, and are graphed in fig. 3, the women being shown in black. The ·4 in the first column of the table means ·4 and less than ·5, the ·5 means ·5 and less than ·6, etc.

Number specifying Red Blindness.	Men.	Women.	Total.
·4	1	0	1
·5	6	1	7
·6	25	5	30
·7	29	5	34
·8	57	10	67
·9	118	13	131
1·0	115	5	120
1·1	77	5	82
1·2	39	7	46
1·3	32	3	35
1·4	20	0	20
1·5	19	1	20
1·6	8	1	9
1·7	1	0	1
2·2	1	0	1
Total	548	56	604

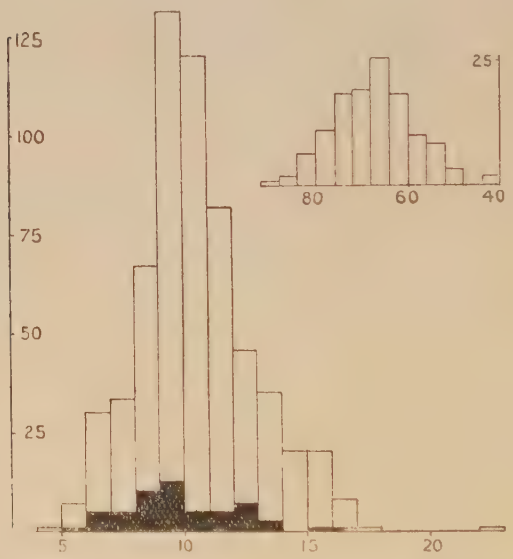
The distribution is slightly asymmetrical, and there is one very red-blind observer very far out by himself, but we cannot definitely say, owing to the asymmetry, since there is only one, that he does not belong to the main group. In the red-green test and Rayleigh tests, described in previous papers*, 13 men out of 835 and 29 men out of 423 respectively fell outside the continuous distribution. As the number of men in the present case is 548, the similar

* Houstoun and Dunlop, Phil. Mag. xli. p. 186 (1921); Houstoun, Proc. Roy. Soc. A cii. p. 353 (1922).

proportion would be 8·5 or 37·6. So it is clear that conditions are different here; so far as the present survey is concerned, normal variation covers everything.

Our results agree with those of Coblentz and Emerson as well as can be expected. Figs. 2 and 3 represent, of course, the variation of different quantities, but quantities that might certainly be expected to vary together.

Fig. 3. Fig. 2.



In fig. 3 abscissæ represent insensibility to red light, ordinates number of observers. The decimal point has been omitted in the scale of abscissæ. In fig. 2 abscissæ represent sensibility to red light.

As the distribution was a random one, a number of the men, probably about 20, would be classed as colour blind. No systematic attempt was made to pick these out. But certain definite statements can be made about some of the observers.

First of all, the observer least sensitive to red, the man on the extreme right separated from the others by a large gap, has perfectly normal colour vision. He is an accurate and experienced observer, has made every one of the tests we have carried out in the past six years, and has never before shown any peculiarity: his performance in the

present test came as a surprise. His case is described fully as an example of normal colour vision in a recent paper*, and yet the average man is about 18 times as sensitive to red light as he is. Edridge-Green† states that "a person with a considerable shortening of the red end of the spectrum will look at a red light (which is so dazzlingly bright to a normal sighted person as to make his eyes ache after looking at it closely for a few seconds) at a distance of a few inches, and remark that there is nothing visible, and that the whole is absolutely black." This is much more extreme than the case recorded above.

Again, the next ten observers, 1·6 and less than 1·8, on being questioned, stated that, as far as they themselves were aware, their colour vision was normal; also four observers who were known to be colour blind before making the test were placed respectively at ·87, ·55, ·62 and above 1·7. The fourth of these is not entered in the table and diagram, as he was a selected case, and did not belong to the random distribution that made the test.

There is no doubt whatever that certain individuals can be very insensitive to red and yet have excellent colour vision. It also seems true, from our own observations and the observations of others, that colour-blind individuals can, in certain cases, have normal sensibility in the red, but the probability is that they will also be abnormal in this respect.

We cannot assume that the distribution in fig. 3 is due to the superposition of a large symmetrical curve and a smaller one on its right due to the "red blind" of the original Young-Helmholtz theory, because most of the observers situated in this region have perfectly good colour vision.

Some of the "scatter" exhibited in the diagram is due to error of observation. A rough estimate of the influence of this cause was made in the following manner:—Two readings were, of course, recorded for each of the observers. The differences of these were taken, squared, added, and the root mean square extracted. The result was ·128. If we divide this by $\sqrt{2}$ we obtain ·09 as the mean error of a single determination of the average observer. Hence, from an inspection of the diagram, probably one-quarter or one-fifth of the scatter of the main group was due to error of observation. The remainder is due to a real difference in the physiological equipment of the observer.

* Proc. Roy. Soc. Edin. xlii. p. 75 (1922).
† 'The Physiology of Vision,' p. 211.

§3. In a paper entitled "A Statistical Survey of the Colour Vision of 1000 Students," which has already been cited, Miss Dunlop and Dr. Houstoun give, on p. 191, the results of certain tests on the colour vision of 165 women students. Miss Dunlop tested another 148 women students after this paper was published, and it is now desired to substitute the following table for the second part of the table on p. 191:—

Microscope Test. Results for 313 *Women Students.*

Range.	Black.	Red-Green.	Blue-Green.	Range.	Black.	Red-Green.	Blue-Green
1 & < 2	0	1	0	13 & < 14	58	41	38
2 & < 3	1	0	0	14 & < 15	30	36	48
3 & < 4	1	0	0	15 & < 16	15	43	50
4 & < 5	0	0	1	16 & < 17	9	36	33
5 & < 6	1	1	3	17 & < 18	2	34	24
6 & < 7	3	3	2	18 & < 19	2	24	20
7 & < 8	4	4	3	19 & < 20	0	11	24
8 & < 9	8	4	2	20 & < 21	0	6	11
9 & < 10	14	4	4	21 & < 22	0	5	1
10 & < 11	38	17	10	22 & < 23	1	1	3
11 & < 12	50	12	12	23 & < 24	1	1	3
12 & < 13	75	29	18	24 & < 25	0	0	3

When an observer makes the microscope test, owing to fatigue, inattention, differences in health, or other accidental variations he does not always obtain the same results. Some of the "scatter" exhibited in the diagrams of the former paper is due to this cause, and not to difference in the physiological equipment of the observers. In the former paper no indication is given of how much of the scatter is due to this cause, but as each observer made two readings, it is possible in the manner described above to show that, in the case of the red-green test, when the 13 observers in the "tail" are omitted, about four-tenths of the scatter is due to error of observation, both in the case of the men and the women. The women do not give more consistent results than the men; also the observers on both sides of the main curve are equally erratic.

July 1923.

LXXIX. *The Effect of the Rolling of a Ship on the Readings of a Marine Mercury Barometer.* *By* M. A. Giblett, *M.Sc., Assistant Superintendent, Forecast Division, Meteorological Office, Air Ministry* *.

1. THIS paper is devoted to an investigation of the effect on the readings of a marine mercury barometer of a periodic motion of the point of support, such as would be associated with the rolling (or pitching) of a ship. It arose from some comparative readings of a marine Kew pattern mercury barometer and an Admiralty Mark I. aneroid barometer, made on a trawler on the North Sea, from which it is deduced below that at times the mercury in the former was depressed below its true level, remaining so, without "pumping," for considerable periods. It is not found possible to explain this behaviour on the basis of the results of previous investigations into the errors to which a mercury barometer is subject at sea, which are summarized in paragraph 2. These do not seem to have included the effect of the forced oscillations of the instrument due to rolling, which is here shown to be of the kind observed.

2. In a paper contributed to the 'Quarterly Journal of the Royal Meteorological Society' in 1908, Gold showed that the effect of a vertical simple harmonic motion of the point of support of the instrument, such as would arise from the heaving of the ship, is an oscillation of the mercury about its *true* height as mean, with an amplitude depending on the constriction of the tube. The greater the "lagging time" of the instrument the smaller the amplitude, which, however, may still be appreciable for instruments with the normal lagging time of 4 minutes. In a case investigated, the range of the oscillation was of the order of two-thirds of a millibar, "pumping" thus being still sufficiently great to introduce uncertainties into the readings. Another source of error investigated was the free oscillation of the barometer about its point of support, the effect of which is to cause the mercury to oscillate about a mean level differing from the true height of the column. Whether the mean displacement of the mercury is one of depression or elevation depends on whether the centrifugal force on the mercury, due to the swing, is sufficient to overcome the tendency of the mercury to run up the tube when the latter is inclined to the vertical, and this depends on the construction of the barometer. For

* Communicated by the Director, The Meteorological Office, Air Ministry.

Kew pattern barometers of the kind commonly in use the result is an *elevation* of the mercury above its true height by a mean amount which may, for small oscillations of the instrument, be of the order of a millibar. (The amplitude of the oscillation about this new mean height is in this case rendered negligible by the constricted tube.) Duffield and Littlewood * have since shown how an adjustment may be made to a Kew pattern marine barometer so that the two opposing influences above may be made to neutralize one another for small amplitudes of swing, thus eliminating this source of error.

Now, in the paper already cited, Gold made a comparison of barometer readings taken on ships crossing a line between Falmouth and Brest with values deduced from readings at these two places, with the result that in practically all cases the ships' observations were markedly *lower* than would be expected from the shore readings, by an amount varying between wide limits. A partial explanation of this was found in the further source of error arising at sea from greater exposure to wind, which may exert a considerable suction effect if, say, the barometer is located in the chart-room with the lee-door open. But, even allowing for this, there remained on an average a further depression of the mercury of the order of two-thirds of a millibar to be accounted for, and it was suggested that the practice of reading the lowest instead of the average height of the mercury when "pumping" would contribute to this. The present results show that the effect of rolling would also contribute.

3. The observations were taken incidentally during an official cruise on the North Sea on the 'George Bligh,' a steam trawler of the 'Mersey' type, belonging to the Ministry of Agriculture and Fisheries. The mercury barometer was set up in a deck-house approximately amidships, and was placed centrally as regards distance from the ship's sides. Its cistern was about 8 feet above water level under calm conditions. The aneroid was placed close to the mercury barometer, at cistern level, and only set roughly. It was very unfortunate that circumstances did not permit of a set of comparative readings being taken immediately before leaving port and again immediately after arrival back, but a comparison was made on land in November of the same year. The pairs of readings at sea in August numbered 61, spread irregularly over 8 days, and the differences ranged over 2·6 mb. about the mean difference

* Phil. Mag. xlii. pp. 166–173, July 1921.

0·47 mb.* The pairs of readings on land in November numbered 43, spread irregularly over a period of more than a month, during which pressure varied through the same part of the scale as at sea in August. The difference on land ranged over 0·8 mb. only, about a mean of 1·1 mb., the standard deviation being 0·21 mb. If we could assume that the result of this comparison would have been the same had it been made in August immediately on reaching port, then we could infer that at sea there was a considerable number of differences outside this range on the negative side, such as could only be explained by some factor, operative at sea only, depressing the mercury in the one barometer without affecting the aneroid, or causing the latter to read too high without affecting the mercury barometer. This is clear from the following table showing the frequency distribution of the differences at sea :—

TABLE I.

(Showing frequency distribution of differences between readings of mercury and aneroid barometer at sea *.)

Difference (mb.)	−1·0 to −0·8	−0·7 to −0·5	−0·4 to −0·2	−0·1 to 0·1	0·2 to 0·4	0·5 to 0·7	0·8 to 1·0	1·1 to 1·3	1·4 to 1·6
Frequency	2	1	4	4	12	22	9	5	2

(Mean difference 0·47 mb.; range 2·6 mb.)

There is no reason to suppose that the range of 0·8 mb. and the standard deviation of 0·21 mb. would have been appreciably different on land in August, but the mean difference 1·1 mb. might have been, as I cannot be certain that the adjustment of the aneroid was the same. However, two readings taken while at anchor in Bridlington Bay in calm weather can be regarded as having been made under similar conditions to those on land, and the ship was remarkably steady on one occasion at sea. The differences in these cases were 0·7, 0·9, and 0·8 mb. Now the probable error of a mean value from n observations is $\frac{2}{3}\frac{\sigma}{\sqrt{n}}$. Here $n=3$

* A positive difference indicates reading of mercury barometer higher than aneroid.

and $\sigma = 0{\cdot}21$ from the November comparison (based on 43 observations), so that the probable error in accepting 0·8 mb. as the mean about which the differences would have been distributed in a series of observations on land with the August setting of the aneroid is less than 0·1 mb.

Thus the above conclusion is still justified. Now the two factors operating at sea and not on land are the additional exposure to wind and the motion of the ship. Any sustained suction or pressure effect of the wind on the pressure in the deck-house should affect both instruments equally. It is also difficult to see how the motion of the ship could cause the aneroid barometer to read too high, so that it appears probable that the motion of the ship was at times such as to cause a persistent depression of the mercury in the mercury barometer of the order of 1 mb., persistent, because similar differences between the readings of the two instruments occurred in spells, and, further, there was no appreciable "pumping" of the mercury at any time during the cruise. From § 2 it follows that if this was the case it must have arisen from the rolling of the ship, and it remains to investigate the possibility of this.

4. The following notation is employed:—

d, the distance of the point of support of the barometer from the axis about which the ship rolls.

ϕ, the angle of inclination of the ship to the vertical at any instant.

x, the corresponding displacement of the point of support of the barometer. ($x = \phi d$.)

ϕ_m, the maximum value of ϕ.

b, the maximum value of x. ($b = \phi_m d$.)

T, the period of the roll. ($T = 2\pi/n$.)

L, the length of the corresponding simple equivalent pendulum.

θ, the angle of inclination of the barometer to the vertical at any instant.

λ, the maximum value of θ.

τ, the period of the barometer for free oscillations about its point of support. ($\tau = 2\pi/m$.)

l, the length of the corresponding simple equivalent pendulum.

ζ, the distance of the centre of the mercury column from the point of support of the instrument.

R, r, ρ, ratios defined by $R=L/l=T^2/\tau^2$, $r=h/l$, $\rho=\zeta/l$.

h, the true height of the barometer.

z, the actual length of the mercury column at any instant.

h', the mean length of the mercury column while oscillating.

δ, the corresponding relative error in the readings. $\delta=(h-h')/h$.

β, the amplitude of the oscillations of the mercury column.

$1/q$, the "lagging time" of the barometer.

h'', a length greater than h, depending on the constriction of the barometer tube.

5. Let us then examine the effect of a periodic displacement of the point of support P of a marine mercury barometer given by $x=\phi d=b\cos nt$, where d is the distance of P from the axis about which the ship rolls, ϕ is the angle of inclination of the ship at any instant to its equilibrium position,

Fig. 1.

and x is the length of the corresponding circular arc described by P (fig. 1). If θ be the inclination of the instrument at any instant to the vertical, the equation of its motion is

$$l\ddot{\theta}+g\sin\theta+\ddot{x}\cos(\theta+\phi)-\frac{\dot{x}^2}{d}\sin(\theta+\phi)=0,$$

where l is the length of the simple equivalent pendulum for

free oscillations of the barometer about its point of support. If θ and ϕ are small this may be written approximately

$$l\ddot{\theta}+g(1-\ddot{x}^2/gd)\theta+\ddot{x}(1-\ddot{x}^2\phi/d\ddot{x})=0$$

or $$l\ddot{\theta}+g\{1-\phi_m^2(\sin^2 nt)4\pi^2d/\mathrm{T}^2g\}\theta+\ddot{x}(1+\phi_m^2\sin^2 nt)=0,$$

where $\phi_m=b/d=$angle through which ship rolls on either side of the vertical, and T is the period of the roll.

Now $$4\pi^2d/\mathrm{T}^2g=1{\cdot}23d/\mathrm{T}^2$$

(units foot, second), and in practice it would scarcely be necessary to consider cases in which $d/\mathrm{T}^2>1$, so that, ϕ_m being assumed small, the equation of motion may further be written approximately

$$l\ddot{\theta}+g\theta+\ddot{x}=0.$$

Writing $m^2=g/l$, $n^2=g/\mathrm{L}$, $r=b/l$, $\mathrm{R}=\mathrm{L}/l$, and $\lambda=r/(\mathrm{R}-1)$, the solution of this equation may be shown to be

$$\theta=\lambda\cos nt+\mathrm{A}\cos(mt+\epsilon),$$

where A and ϵ are arbitrary constants. The term $\mathrm{A}\cos(mt+\epsilon)$ merely represents a free oscillation of the instrument, of arbitrary amplitude and phase, superimposed on the forced oscillation given by $\lambda\cos nt$. The effect of the free oscillation alone has already been referred to in § 2, and it is our present purpose to consider the forced oscillation given by

$$\theta=\lambda\cos nt,$$

supposing the free oscillation absent.

Now the column of mercury may be regarded as subject to an acceleration along the tube, relative to the instrument, equal to

$$g\cos\theta+\zeta\dot{\theta}^2-\ddot{x}\sin(\theta+\phi)-\frac{\ddot{x}^2}{d}\cos(\theta+\phi),$$

where ζ is the distance of the centre of the column from the point of support. Supposing θ and ϕ small, this may be expressed as a function of t in the form

$$g\left[1+\frac{r^2(\mathrm{R}+2\rho-2)}{4\mathrm{R}(\mathrm{R}-1)^2}+\left\{\frac{r^2(\mathrm{R}-2\rho-2)}{4\mathrm{R}(\mathrm{R}-1)^2}+\phi_m^2\cdot\frac{4\pi^2d}{\mathrm{T}^2g}\right\}\cos 2nt\right],\quad .\quad .\quad (\mathrm{A})$$

where $\rho=\zeta/l$.

Calling the constant part $g+g'$, suppose that under the action of this the mercury assumes a height h' instead of

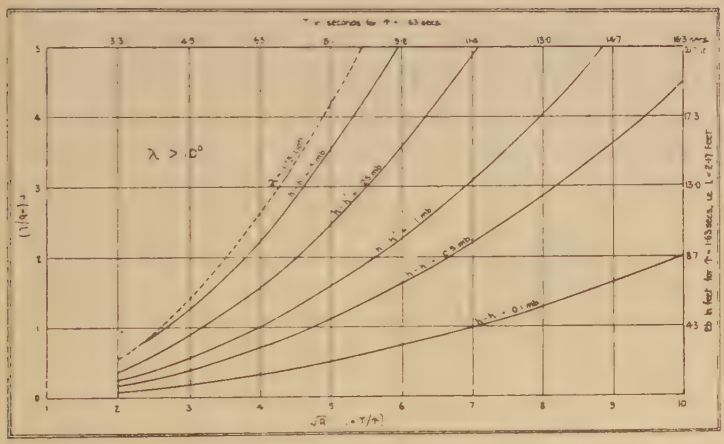
the true height h. Then $hg=h'(g+g')$, so that, g' being small compared with g,

$$\delta=(h-h')/h=g'/g=\frac{r^2(\mathrm{R}+2\rho-2)}{4\mathrm{R}(\mathrm{R}-1)^2}. \quad . \quad . \quad (\mathrm{B})$$

It is evident that the periodic acceleration represented by the third term of (A) above would cause oscillations of the mercury about the mean height h', and, if the amplitude of these were rendered negligible by the constriction of the tube, then the mercury would simply assume a steady height h' instead of h.

6. The relative error, δ, due to the forced oscillation, is given by (B) in terms of the three ratios $\rho\,(=\zeta/l)$, $r\,(=b/l)$, and $\mathrm{R}\,(=\mathrm{L}/l=\mathrm{T}^2/\tau^2)$, where ζ is the distance of the centre

Fig. 2.



Showing the amount of depression of the mercury in a marine mercury barometer, due to the rolling of a ship. (For explanation see § 6.)

of the mercury column from the point of support, l is the length of the simple equivalent pendulum for free oscillations of the barometer about its point of support (period τ), b is half the arc along which the point of support moves, and L is the length of the simple equivalent pendulum for the forced oscillations (period T). We may here also usefully recall that the amplitude of the forced oscillation of the barometer is given by $\lambda=r/(\mathrm{R}-1)$, and that the expression for δ given by (B) has been obtained on the supposition that

λ is small. In fig. 2 abscissæ represent values of $\sqrt{R}$, and ordinates values of r. Pairs of values corresponding to points on the dotted line are such as to make $\lambda=10°$. Attention will be confined to pairs of values of R and r corresponding to points lying to the right of the dotted line, for which $\lambda<10°$.

From (B) it is seen that δ is positive, *i. e.*, the forced oscillation causes a *depression* of the mercury, if $R>2(1-\rho)$. For the barometer employed on the trawler, which may be taken as typical, $\tau=1{\cdot}63$ secs., so that $l=2{\cdot}17$ ft. ζ, by direct measurement, was about 5 ins. for a barometric height of 30 ins., and thus ρ was a fraction like 1/5, varying somewhat with the height of the barometer. Hence $\delta>0$, if $R\,(=T^2/\tau^2)>1{\cdot}6$, *i. e.*, $T>1{\cdot}3\tau$, which would in general be the case at sea. It may here be noted that the adjustment suggested by Duffield and Littlewood (see § 2) is such as to make $\rho=1/2$, in which case the condition for $\delta>0$ would be $R>1$ or $T>\tau$.

The depression of the mercury corresponding to different values of R and r, for $\rho=1/5$, and $h=1000$ mb.*, is indicated in fig. 2 by means of curves along any one of which it is constant.

The curves may also be read in conjunction with the scales for T and $2b$, at the top and on the right of the diagram, in the particular case when $\tau=1{\cdot}63$ seconds (and $\therefore\ l=2{\cdot}17$ feet).

7. The amplitude of the forced oscillations of the mercury may be investigated by adapting to the present case the equation of motion of the mercury as given by Gold † for the case of free oscillations of the barometer. It may then be written

$$h''\ddot{z}+\frac{g}{q}\dot{z}+\left\{g\cos\theta-\ddot{x}\sin(\theta+\phi)-\frac{\dot{x}^2}{d}\cos(\theta+\phi)+\zeta\dot{\theta}^2\right\}z-gh=0,$$

where h'' is a length ($>h$) depending on the constriction of the tube, z is the actual length of the mercury column at any instant, and $1/q$ is the "lagging time" of the barometer.

* The values of the depression would be appreciably the same for other pressures, being changed proportionately.

† *Loc. cit.* § 2.

If θ and ϕ are small this equation may be written approximately

$$h''\ddot{z}+\frac{q}{q}\dot{z}+g\{z-h(1-\delta)\}+gh\delta'\cos 2nt=0,$$

where

$$\delta=\frac{r^2(\mathrm{R}+2\rho-2)}{4\mathrm{R}(\mathrm{R}-1)^2}\quad\text{and}\quad\delta'=\frac{r^2(\mathrm{R}-2\rho-2)}{4\mathrm{R}(\mathrm{R}-1)^2}+\phi_m{}^2\cdot\frac{4\pi^2d}{\mathrm{T}^2g},$$

both being small fractions under the conditions assumed.

The required solution of this is

$$z=h(1-\delta)+\beta\sin(2nt+\psi),$$

where

$$\beta=qh\delta'\{q^2(1-16\pi^2h''/g\mathrm{T}^2)^2+16\pi^2/\mathrm{T}^2\}^{\frac{1}{2}}. \quad . \quad (\mathrm{C})$$

Thus the mercury oscillates about a mean height h' given by

$$h'=h(1-\delta).$$

This gives $\delta=(h-h')/h$, as obtained otherwise in § 5.

8. To evaluate the amplitude (β) of the oscillations of the mercury let us take, following Gold, $h''=5$ ft. and $q=1/240$. Further take $l=2{\cdot}2$ ft. Then in the denominator of (6) the first term may be neglected in comparison with the second, so that approximately

$$\beta=qh\delta'\mathrm{T}/4\pi.$$

Now, from § 7,

$$\delta'=\mathrm{Q}+\phi_m{}^2\cdot 4\pi^2d/g\mathrm{T}^2,$$

where $\mathrm{Q}<\delta$, whence

$$\beta=h\mathrm{Q}\cdot q\mathrm{T}/4\pi+h\pi\, dq/\phi_m{}^2/g\mathrm{T}.$$

But $h\mathrm{Q}<h\delta$, which is the quantity represented in fig. 2, so that evidently the first term may be neglected, being, for values of T considered, only a very small fraction of a millibar. Hence approximately

$$\beta=h\pi\, dq\,\phi_m{}^2/g\mathrm{T}. \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{D})$$

If we suppose $\phi_m<0{\cdot}175$ radian (*i. e.* 10°), and $d<50$ feet, then $\beta<1/1600\,\mathrm{T}$, so that if $\mathrm{T}>5$ secs., say, 2β is less than 0·25 mb., and in general considerably less.

9. The effect of the forced oscillations of the instrument on its readings has been examined above only for small angles of roll of the ship, but it is found that even such may result in a persistent depression of the mercury by an amount which is not negligible. Pumping is, however, not likely to

be marked in such cases. The effect is thus precisely that observed on the North Sea, and the magnitude too is quite in agreement. The conditions encountered on the North Sea were such as to produce a forced oscillation of period 6 to 7 seconds. As a very rough estimate we may take the point of support of the barometer as having been 10 ft. above the axis about which the vessel was rolling. A roll of only 10° each side of the vertical would then imply that the point of support ranged through 3·5 ft. From fig. 2 we see that $T = 6{\cdot}5$ secs. and $2b = 3{\cdot}5$ ft. correspond with a depression of the mercury between 0·5 and 1 mb., which is of the requisite magnitude. Further (D) in § 8 shows that pumping in this case would be negligible.

The period of the forced oscillation in the particular case discussed is small. For longer periods fig. 2 shows that the depression of the mercury is negligibly small unless the point of support of the barometer ranges through a considerable distance. However, the usual practice of placing the barometer for convenience in the chart room favours this, resulting as it does in the instrument being well above the axis about which the ship rolls. For example, the barometer is commonly 30 or 40 feet above the water-plane, and the ship may be considered roughly as rolling about an axis lying between the water-plane and half-draught *. A roll of only 10° on either side of the vertical might accordingly result in the point of support of the barometer ranging through about 14 feet ($2b = 14$), and fig. 2 shows that, corresponding to this, the barometer would read 1 mb. too low if the period were between 11 and 12 seconds, and 0·5 mb. if the period were nearly 14 seconds. It is evident that, while in many cases the effect might be negligible, yet for accurate work it would always be advisable to mount the instrument as near as possible to the axis about which the ship rolls. An interesting case of the different behaviour of two barometers placed in different parts of a ship was mentioned in the discussion of Col. Gold's paper † by Capt. R. C. Warden. Under ordinary weather conditions the instruments agreed, but during the passage of two typhoons in the China Seas their readings differed. It is possible that this was due to the different effects of the forced oscillations of the instruments during the disturbed period, owing to their different locations in the ship.

* 'Admiralty Manual of Seamanship,' 1909, vol. ii. p. 571.
† *Loc. cit.* § 2.

LXXX. *Notices respecting New Books.*

Tables of the Incomplete Γ *Function.* Edited by KARL PEARSON, F.R.S. £2 2*s.* (His Majesty's Stationery Office, Kingsway, W.C. 2.)

SEVERAL important sets of tables have been calculated by the Staff of the Department of Applied Statistics in connexion with the Gamma and similar functions. The difficult and laborious computation of the Incomplete Γ function has also been undertaken by members of this Staff, who have placed those interested in applied mathematics under a great obligation through the completion of these valuable tables. The fact that this volume contains some fifty thousand seven-figure values with second and fourth differences for the two variables, will give some idea of the magnitude of this arduous undertaking, "a painfull work" indeed.

In order to overcome the difficulty arising from the large range of values of the function, the ratio of the Incomplete to the Complete Γ functions is tabulated, restricting the entries between the limits zero and unity: the variable x, in $\int_0^x e^{-x}x^p.\,dx$ is replaced by a variable u connected with x by the relation $u=x/\sqrt{p+1}$.

The introduction sets out fully the methods of using the tables and computing intermediate values of the function by the various interpolation formulæ. Although thousands of entries are given in this volume, these interpolations may be considerable, but in comparison with the labour required in building up the tables, the computer's work is well-nigh insignificant.

It is encouraging to note that, on the recommendation of the Advisory Council, the Department of Scientific and Industrial Research have undertaken the publication of this work: the publication of tables of other functions related to the Gamma function also deserves the real assistance which has been accorded to this volume.

Mathematical Tables. By G. H. BRYAN, Sc.D., F.R.S. (Macmillan & Co.) 3*s.* 6*d.*

IN addition to the tables of the six trigonometric functions and their logarithms, which form the greater part of this book, Prof. Bryan has included the Bordered Antilogarithm Table, giving logarithms, antilogarithms, and logarithms of reciprocals. The entries in this short and compact table are to five places for smaller values of the logarithm, with proportional differences of one or two digits. The table of squares can be employed to better effect for the calculation of square roots than by the use of logarithms. In the methods of using the tables, special attention is given to the Antilogarithm Table, the most important feature of this collection.

LXXXI. *Proceedings of Learned Societies.*

GEOLOGICAL SOCIETY.

[Continued from p. 576.]

April 18th, 1923.—Prof. A. C. Seward, Sc.D., F.R.S., President, and, afterwards, Dr. Herbert H. Thomas, M.A., Vice-President, in the Chair.

THE following communication was read :—

'The Structure of the Bowmore-Portaskaig District of Islay.' By John Frederick Norman Green, B.A., F.G.S.

The island occupies a critical position in connexion with present controversies in Highland geology, because (*a*) rocks of the Eastern Highland type there approach the North-West Highland rocks ; (*b*) in it cross the lines of work of researchers of different schools ; and (*c*) over much of the island the original characters of the rocks are exceptionally preserved.

All observers agree that the western peninsula (the Rhinns) is composed of Lewisian gneiss overlain by Torridonian. The author deals only with the northern and central parts of the main island.

In this part the Geological Survey, in a memoir published in 1907, regarded a dolomitic group as probably Cambrian, preserved in synclines of older quartzite, at the base of which a conglomerate (Portaskaig Conglomerate) with granite (nordmarkite) and other boulders, rested upon limestone (Islay Limestone) and dark slates (Mull of Oa Phyllites) : the whole series being brought by a great thrust (Loch Skerrols Thrust) over Torridonian (Bowmore Sandstone).

Later, Mr. E. B. Bailey, in a paper read in 1916, held that the dolomitic group was intercalated between a 'Lower Quartzite' connected with the Conglomerate and an 'Upper Quartzite' (the Islay Quartzite), confirming otherwise on the whole the Survey views, and correlating the Loch Skerrols Thrust with the Moine Thrust ; but Mr. Barrow (in discussion) considered that the thrust had no existence, that the Bowmore Sandstone was not separable from the rocks east of the supposed thrust, and that the sequence should be inverted, the limestone overlying the conglomerate, which in turn overlay the dolomitic group and the Islay Quartzite.

The author's attempt to find the correct interpretation has necessitated a quantitative definition of the term 'quartzite,' which has previously been used to cover a wide range of rocks in Islay. For the purpose of this paper a limit of 10 per cent. of felspar has been adopted, more highly felspathic rocks being termed 'arkose.' On this definition the Islay 'Upper' Quartzite is throughout true quartzite ; but the so-called 'Lower Quartzite'

is not quartzitic, being composed of arkoses and greywacke-slates identical with the matrix of the Portaskaig Conglomerate. As much of the latter horizon is not conglomeratic, and no dividing line can be drawn between the two, they have been grouped together as the Portaskaig Beds. Thus the dolomitic group intervenes between the Portaskaig Beds and the Islay Quartzite.

It is next shown that the supposedly Torridonian Bowmore Sandstone consists of arkoses and flags, the former being identical in all respects with the matrix of the Portaskaig Beds, and containing in places similar pebbles, including the characteristic nordmarkite. The flags, which, owing to isoclinal folding, have apparently an enormous thickness, are identical in minute detail with certain siliceo-argillaceous flags that always occur in the dolomitic group next to the Islay Quartzite. They are termed the Bowmore Flags.

Thus there is no change of facies at the supposed thrust, and an examination of its presumed line of outcrop proves that the rocks are folded up without disruption, except for some shearing on the reversed limbs of overfolds. The Loch Skerrols Thrust is non-existent.

The relations of the dolomitic group are then investigated, and it is found, first, that the Bowmore Flags are perfectly conformable to the white edge (a well-marked horizon) of the Islay Quartzite; but secondly that they, and the dolomitic flags associated with them, are partly or wholly cut out in places by the Portaskaig Beds. It is inferred that the latter are younger than the dolomitic group, from which they derive material.

The sequence within the dolomitic group appears to be reversed along Islay Sound. It is shown, however, that this misleading section can be explained by a combination of erosion and isoclinal folding.

These conclusions necessitate a re-examination of the evidence at Beannan Dubh, which has been taken to prove that the Portaskaig Beds are younger than the Limestone, and therefore older than the dolomitic group. Reasons are adduced for regarding the supposed disposition of strata in this neighbourhood in nearly horizontal beds as deceptive, the rocks actually lying in isoclinal folds with low dip, by which the Portaskaig Conglomerate is brought up in anticlines. On Beannan Dubh a dolomite-breccia resting upon a thin band of dolomite within the Portaskaig Beds, which has been believed to prove superposition, is regarded by the author as a desiccation-breccia and shown to be repeated in a reversed position. The evidence of solution-hollows is considered to favour true superposition of the dolomite-band on the breccia, supporting the view that the Limestone is younger than the Portaskaig Beds.

Thus all lines of evidence converge to prove the order of succession proposed by Mr. Barrow, and the structure of Islay is synclinal, not anticlinal. Only one system of folding is required to explain the facts.

LXXXII. *Intelligence and Miscellaneous Articles.*

ON IONIC OSCILLATIONS IN THE STRIATED GLOW DISCHARGE.

To the Editors of the Philosophical Magazine.

GENTLEMEN,—

IN the May number of the Philosophical Magazine appears a paper by Dr. Appleton and Mr. West dealing with the discontinuous nature of the vacuum-tube discharge under some conditions and the consequent radiation therefrom.

I am venturing to point out, with your permission, that the literature of the subject is not without references to this phenomenon, which, indeed, to those who have worked with discharge-tubes running under steady voltage conditions, must be familiar.

As just two early examples may I cite

I. *M. Cantor*, Wied. Annal. 67, ii. (1899).

This investigator performed an experiment strikingly similar to that of Messrs. Appleton and West. He showed quite conclusively that electric waves were radiated from a discharge-tube under certain conditions of steady voltage supply, and specifically mentions a number of precautions designed to eliminate all possibility of the mere starting of the discharge producing the radiation.

II. *Rieche*, Ann. d. Phys. iv. (1901).

This investigator looked into the matter from the point of view of the variation of the current and luminosity in the tube, by the use of a telephone and a rotating-mirror respectively. By this time, the phenomenon of periodic intermittence of the discharge (tube run from a battery of accumulators under certain conditions) appears to be a commonplace, for he remarks :—"bei hoheren Drucken das bekannte Flackern das Glimmlichtes eintret. Ihre Anzahl in der Sekunde schwenkte von 50 bis über 2000."

The frequency was determined in a most ingenious way by observing simultaneously in the rotating mirror the reflexion of the discharge-tube and that of a sensitive flame jumping with a known and variable frequency.

Yours faithfully,

R. WHIDDINGTON.

The Physics Laboratories,
The University, Leeds.
25th June, 1923.